

CSC2421: Online and Other Myopic Algorithms

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Lecture 8

Announcements

- Thanksgiving is next Monday so no class.
- I have 6 people so far who have sent me their project suggestions. I should have acknowledged that I received your suggestions. If you sent me your suggestions but didn't get a reply from me, please resend.

I repeat that my taste is for problems where there are different special cases or extensions possible rather than very specific constant gaps in the known positive and negative results but that is my taste. Of course, I again repeat that solving such gaps could be very interesting given how it is solved and given the importance and history of the problem.

Today's agenda

- Finish up discussion of Volume 2 Open Problems (before OpenAI or some LLM solves them all).

OpenAI announces solutions to hundreds of problems

One of the research areas is Theoretical Computer Science (TCS). Most of the results in this area are complexity theory results.

However, they do announce the solution to the Randomized k-Server Conjecture. They claim to show that the competitive ratio is $O(\log^2(k + 1))$. I have posted that paper on the web page.

This still leaves open the ratio for some specific metric spaces of interest such as the line and tree metrics.

They also proved a result concerning the taking of a single sample for the Prophet Inequalities problem.

The OpenAI Catalogue

MATHEMATICS

OPENAI

OpenAI Research Catalog

372 result families in 722 manuscripts

October 6, 2026

An overview of the [manuscript collection](#). Results are grouped by subject; entry numbers follow the catalog order and do not indicate a ranking. PDF links follow each summary.

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Open problems: Volume 2, Chapter 5 continued

Open problem 6 asks whether or not there is a constant competitive algorithm for the Throughput Maximization with C Benevolent weights in the real-time model with revoking.

Open problem 7 concerns the throughput problem for m machines.

Open problems 8 and 9 concern the QoS problem.

Open problem 10 concerns the Metric Matching with delays problem. Pretty much, in almost every setting of this problem, there are open problems.

There are many problems which have only been studied in the online model that could be recast in the real-time model with delays.

Open problems: Volume 2, Chapter 6

We return to the online (not real-time) model.

The first three open problems concern the interval selection problem in the online model with revoking.

The first open problem is to determine the randomized competitive ratio for adversarial sequence. The current upper bound is 6 and there is a weak lower bound (negative result) that shows that there is no optimum ratio = 1 algorithm.

The second interval selection problem concerns the ratio for deterministic algorithms with ROA arrivals for the unweighted case. , I have an unpublished paper with Chris Karavasilis and David Zhang where we claimed a 2.5 ratio. We asked Claude to verify the proof but Claude thinks our proof is problematic. Claude used our claim to prove an even better 2-competitive algorithm. The lower bound Claude gives is $4/3$. We have to check the proof.

Open problems: Volume 2, Chapter 6

The third of the interval selection problems is with respect to deterministic algorithms with random order arrivals for the case of proportional weights.

The fourth open problem is about the General Knapsack problem with random order arrivals. We are interested in the online models with and without revoking. I have another paper with Chris and David (which Claude thinks is OK) where we show how to de-randomize 1-bit random algorithms in the ROA model with revoking. I plan to discuss de-randomization of barely random algorithms in a future class.

The fifth open problem asks you to define a two (or multi) pass model(s) for the unweighted (one-sided) bipartite matching problem. What are natural restrictions on what can be remembered between passes?

Open problems: Volume 2, Chapter 8

The first open question concerns the proportional weights interval scheduling problem on $m > 1$ machines. Perhaps surprisingly, we get better ratios for $m > 1$.

A specific question within the second open problem is whether we can obtain a constant competitive algorithm for C-Benevolent weight functions noting that proportional weights are a special case of C-Benevolent weights.

The final open problem is rather vague. Given an assumption on the graph representation model, we would like to have an model for what we call greedy algorithms when considering classes of graphs and corresponding graph problems.

For example, consider the VAM-PH and VAM-FI graph input models defined in Chapter 7 of Volume 1. We would like to re-consider graph problems (as in Volume 1, Chapter 7) in models such as the priority model and other “natural” algorithmic models we might define.

Greedy and Other Myopic Algorithms for Graph Problems

In particular, consider the VAM-FI graph input model.

The class of d -inductive graphs and the larger class of d -inductively independent graphs are defined by the existence of vertex orderings. Namely, a graph is d -inductive (respectively, d -inductively independent) if there is an ordering of the vertices $v_1, v_2, \dots, v_n$ such that $NBHD(v_i \cap \{v_{i+1}, \dots, v_n\})$ has degree at most d (respectively, has at most d independent vertices). See my paper (on the web page) with Yuli Ye called Elimination Graphs.

As an example, the class of $d = 1$ inductively independent graphs (i.e., *chordal* graphs) contains the class of interval graphs.

Once we have the appropriate ordering, we can obtain a d approximation for the unweighted Maximum Independent Set (MIS) problem. And, given the reverse of such an ordering, we can obtain a d -approximation for the graph colouring problem.