

CSC2421: Online and Other Myopic Algorithms

Fall 2026

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October 5, 2026

Lecture 7

Announcements

- Next Monday is Thanksgiving, so no class.
- Please send me your suggestions for a possible individual project and for a group project. You can suggest open problems for both the individual and the group project but your actual individual and group project must be different.
- Given changes we are making to reflect the recent progress and current state of problems, I produced a new version of Volume 2 and reposted it in Quercus. I will keep both versions on Quercus until we finish the discussion of open problems. Then I will just keep the current version which I will keep updating.

Today's agenda

We will continue discussing the open problems in Volume 2 (in the version now in Quercus).

Open problems: Volume 2, Chapter 2

This chapter concerns stochastic input models and, in particular, the ROM, i.i.d, i.d., and Markov input models. We have only listed a very small number of problems relative to all the problems one could specify. As I already stated, I particularly like the ROM model.

As I mentioned in our third class, Open problem 1 has now been substantially solved in a recent September, 2026 arXiv paper. Singla (using AI) gives a constant $\frac{1}{4}$ competitive randomized algorithm for the matroid secretary problem. Huang quickly followed providing an optimal $\frac{1}{e}$ competitive ratio for the matroid secretary problem.

The second open problem concerns the well-studied (but still not fully resolved) Prophet Secretary problem for the special case of i.i.d. input sequences. There is an open problem for Prophet Inequalities with i.i.d. input sequences case when the distribution is *unknown* and we allow $O(n)$ samples. With $O(n^2)$ samples, the distribution becomes known.

Open Problems for Chapter 2 continued

The third open problem concerns unweighted bipartite matching in the known i.i.d. input model with non-integral expected arrivals.

Open problem 4 calls attention to the fact that while the randomized $\rho = 1 - \frac{1}{e}$ -competitive Ranking algorithm for the unweighted bipartite matching problem with adversarial input arrivals implies the same ρ ratio for a deterministic algorithm in the ROA model, this leave open whether or not $1 - \frac{1}{e}$ is the best deterministic ratio in the ROA model.

The fifth open problem concerns the uncapacitated facility location problem in the ROA model.

More open problems for Chapter 2

The optimal ratio for the Submodular Welfare Maximization (SWM) problem with known i.i.d. input sequences has recently been solved by Udwani and Wajc. They show that for *known* distributions, the $1 - \frac{1}{e}$ ratio cannot be improved even for randomized algorithms.

It was already known that there is $1 - \frac{1}{e}$ competitive algorithm in the *unknown* i.i.d input model so that we now have the same optimal ratio for both known and unknown distributions for the SWM problem.

The seventh problem in the updated text):

With regard to proving inapproximations for algorithms in the ROA model, one general approach is to prove inapproximations for the i.i.d input model. Are there any other general approaches for establishing inapproximations in the ROA model? In particular, what is the optimal competitive ratio ρ_{ROM} for the SWM problem. The current state for the SWM problem is as follows:

The ratio for adversarial input sequences $= \frac{1}{2} < \rho_{ROM} \leq 1 - \frac{1}{e} =$ the ratio for known i.i.d and unknown input sequences.

Another question regarding deterministic ROM algorithms

The following question was not yet stated in the text: Is there a contrived or natural online problem that has a randomized ρ -competitive algorithm but does not have a deterministic ρ -competitive ratio in the ROA input model.

We have problems for which we can obtain a better ratio for a deterministic algorithm in the ROA model than one can obtain for randomized algorithms with adversarial input arrivals (e.g., the secretary problem).

What if deterministic algorithms with ROA inputs are always at least as good (wrt the competitive ratio) as randomized algorithms with adversarial input sequences? Such a result could be proved non-constructively (which is the nature of the result showing that a ρ -competitive randomized algorithm against an adaptive offline adversary implies a ρ -competitive deterministic algorithm). On the other hand, what would a constructive proof look like?

Open problems: Volume 2, Chapter 5

The first five open problem concern various questions regarding the unweighted and weighted Throughput Maximization problem in the real-time model with restarting. The gap between the known negative and positive competitive results is small for deterministic algorithms. For randomized algorithms, the basic question is how much can randomization help for any of the instances of Throughput Maximization.

We consider the weighted Throughput problem for the special cases of C-Benevolent and D-Benevolent weights, and the case of arbitrary weights but only a single processing time. As far as I know, Throughput Maximization with a small number of processing times has not been considered.

Problem five asks about equal slack.

Open problems: Volume 2, Chapter 5 continued

Open problem 6 asks whether or not there is a constant competitive algorithm for the Throughput Maximization with C Benevolent weights in the real-time model with revoking.

Open problem 7 concerns the throughput problem for m machines.

Open problems 8 and 9 concern the QoS problem.

Open problem 10 concerns the Metric Matching with delays problem. Pretty much, in almost every setting of this problem, there are open problems.

There are many many problems which have only been studied in the online model that could be recast in the real-time model with delays.