

# **CSC2421: Online and Other Myopic Algorithms**

## **Fall 2026**

Allan Borodin

September 28, 2026

# Lecture 5

## Announcements

Enzo Mutiso had a note on Piazza suggesting a post for organizing 2 and 3 person teams for the course assignment. Seems like a great idea. I am hoping that the quercus list is the final enrollment. There are 13 students on the Quercus list. If anyone is not registered but still wants to team up, I am OK with that.

## **Today's agenda**

Walk through some open problems in Volumes 1 and 2. In doing so, I will be defining problems and sometimes saying why I am currently interested in a few of these open problems.

# Some specific open problems in Volume 1

## Chapter 2:

We only have listed three open problems for this chapter. I don't consider narrowing the gap between the known negative and positive results here to be that important. I consider open problems 2 and 3 to be examples where the interest would be mainly if one could obtain an optimal or near-optimal result for these classic problems.

I just want to point out the relation between the time series problem and the secretary problem. Namely both are trying to pick a single winner. In the time series problem the input sequence is adversarial whereas in the secretary problem the input sequence is random order.

In later chapters (in both volumes), there are some open problems where we generalize a problem from adversarial order to random order. Similarly, we can generalize an online problem to the same problem with priority orderings or best orders.

## Open problems: Chapter 3

Open problem 1 seems like a problem where we should know the answer but it isn't clear to me what the answer is. It might be in the literature somewhere. However, be careful since some authors define the unweighted knapsack problem to what we believe should be called the proportional weight knapsack problem.

We can also consider a problem related to Open problem 2; namely extend the proportional weighted result to C-Benevolent weights as defined in chapter 5 of volume 2.

Open question 3 can be more generally considered for all randomized competitive ratios which mainly provide expected ratios and could perform very poorly on some sets of inputs.

We like Open question 4. It might be false in the generality we are asking. So maybe the question could be restricted to covering and packing problems. This question relates to a question in Volume 2 where we consider the Random Order Arrival (ROA or ROM) model.

## Open problems: Chapter 4

The first four open problems are all of the form "close the gap between the best positive and negative results. The interest in these problems depends on how much you like the problem and whether or not you can identify the optimal ratio. I personally am less interested in Open problem 2 since I am less interested in the paid model. Just a matter of taste.

Open problem 5 is a significant problem given the gap and the interest in the Steiner tree problem.

## Open problems: Chapter 5

Open problems 1 and 2 both concern the randomized  $k$ -server conjecture which we might now state as “Does there exist a  $O(\log^2 k)$  (or even  $\text{polylog}(k)$ ) randomized competitive ratio for every metric space. The paper stating that the “ $k$  server conjecture is false” was for the conjecture that there exists a  $O(\log k)$  randomized competitive algorithm for all metric spaces.

In particular, we do not seem to know if there is a  $o(k)$  randomized competitive algorithm for the infinite line.

Open question 3 has been solved as mentioned last week. This is a big deal given the important role that the  $k$ -server conjecture has played in the development of online algorithms.

I am personally not that interested in Open question 4. Again, this is a matter of taste. And whenever I say taste, I should emphasize that if the method of proof is novel then I would be very interested especially if the method might be applicable more generally. .

## Open problems: Chapter 6

Online learning is an area of research I am not that familiar with and here I would first suggest trying to find out what might be the most interesting questions for expert learning and bandit problems. You can ask an LLM what it thinks are the most interesting open questions.

Again since I am not as certain about the status of problems in this area, it could be that Open questions 1 and 2 are already solved,

Open question 3 extends the bandits framework to the random order model discussed in Volume 2. But it is a natural extension that again I wonder if it has been looked at and, if not, why not? As you might already tell bhy now, I am interested in the ROA model.

# Open problems: Chapter 7

## Note:

Open question 1 has recently been solved by Tang and Zhang (arXiv August, 2026) who show that there is no randomized algorithm for the vertex cover problem (with edge arrivals) that achieves a competitive ratio less than 2. Hence the optimum ratio is 2.

Open question 2 is an instance of the general question as to the tradeoff between the number of random bits (or more generally, the amount of entropy in the random source) vs performance (e.g., the competitive ratio). Here the specific question is the number of random bits vs the competitive ratio for the unweighted one-sided bipartite max matching (BMM) problem. In class, I will define the problem and give some of the history and importance of the unweighted and weighted BMM problem.

In particular, the randomized Karp, Vazirani and Vazirani [Ranking algorithm](#) is a seminal paper in competitive analysis.

## Chapter 7 Open Problems Continued

Open problems 3,4,5 concern the min cost perfect (or maximum) matching problem. These are all good research questions that I am sure many people are trying to solve with the help of LLMs.

Problems 6,7,8 all involve min colouring graphs for restricted classes of graphs.

Problem 6 seems like the kind of problem I would be less interested in as it is about narrowing the gap for a particular algorithm (First Fit colouring of interval graphs). But First Fit is so natural that it motivates me (and many others).

Problems 7 and 8 relate to the wide gap in colouring a  $k$  colorable graph.

We ended the class on this slide. We will continue next class with more of the text open problems.