CSC 2420 Spring 2015, Assignment 3
Due: Now extended to Tuesday April 7

NOTE: This assignment should be done by yourself. It is certainly preferable for you to solve the questions without consulting a published source. However, if you are using a published source then you must specify the source and you should try to improve upon the presentation of the result.
NOTE: Do not worry if you cannot solve every question. Do the best you can, If you have some useful partial ideas then mention them. But if you do not have any ideas on a given question, I prefer that you just leave the question blank. I award 20% of the value of the question for the admission "I do not know how to answer this question". That is, I credit people for knowing what they do not know.

1. Using the framework of a "graph search" as defined by Professor Corneil in his lectures, show how depth first search fits within that framework.

2. Fill in the proof for showing that 2-SAT can be computed in deterministic polynomial time. How much space is required to compute 2-SAT?

3. Consider the weighted max-di-cut problem. That is, we are given an edge weighted directed graph $G = (V, E)$ with weight function $w : E \rightarrow \Re^{\geq 0}$ and the goal is find a subset of vertices $S \subseteq V$ so as to maximize $f(S) = \sum_{(u,v) \in E, u \in S, v \in V \setminus S} w(u, v)$.

   (a) Show that f is a non monotone submodular set function.

   (b) What is the expected approximation ratio for the naive randomized algorithm? That is, independently place each vertex in S with probability 1/2.

   (c) Consider the de-randomization (by the method of conditional expectations) of the naive randomized algorithm. Express the derandomized algorithm as a deterministic greedy algorithm.

   (d) (Bonus). This is a case where the derandomization of the naive randomized algorithm turns out to have a better approximation than the expected approximation bound of the randomized algorithm. If you can show an improved bound (without finding it in the literature) than this will count as a bonus question. For consistency lets express approximations heree as fractions.

4. Complete the proof of Lemma II.2 in the Buchbinder et al paper. (I have a link to the paper on the course web page.) That is, give the missing part of the proof when $a_i < b_i$.

5. We are given a degree bound d and query access to a table for a function $f : Q \to Q$; namely given $\{(x_1, f(x_1), \ldots, (x_n, f(x_n)\}$ we can access any $(x_i, f(x_i))$ in one query.

   We consider the following question for $d << n$: We want to test if the partial table f is equal to a degree d polynomial p or if it is "far-away" from any degree d polynomial $p()$ where by far-away we mean that $f(x_i) \neq p(x_i)$ for at least n/d of the points given in the table. Provide a randomized 1-sided error algorithm that will make $O(d)$ queries, run in time polynomial in d, always returning p if it exists and with probability $\geq \delta$ will determine that f is far-away from any degree d polynomial. If neither condition is true, the algorithm can give any answer.

   Analyze the probability δ that can be achieved in terms of the number of queries used.