CSC 2420S 2012, Assignment 2
Due: March 26

1. Prove Lemma 1.2 in Lecture 6.

2. Let $G = (V, E)$ be a node weighted graph where $w(v)$ denotes the weight of $v \in V$, $w(S)$ denotes $\sum_{v_i \in S} w_i$ and $Nbhd(v) = \{v' | (v, v') \in E\}$ is the set of adjacent vertices (the neighbourhood) of v. Consider the following local search algorithm for the weighted max independent set (WMIS) problem.

   $S := \emptyset$
   While $\exists v \in V$ such that $w(S + \{v\} - Nbdh(v)) > w(S)$
       $S := S + \{v\} - Nbhd(v)$
   End While

   Show that this local search algorithm is a k-approximation algorithm when G is a $k + 1$-claw free graph. Show that the bound is tight.

   Do not worry about the convergence time. Essentially show that the locality gap is at most k.

3. Let $w()$ be a monotone, normalized submodular function. Show that the local search algorithm in question 2 is a $k + 1$ approximation for maximizing $w(S)$ subject to S being an independent set in a $k + 1$-claw free graph.

   Again as in question 2, do not worry about convergence time.

4. Consider a set of m linear equations defined over the field GF_2 (the field of 2 elements) by $Ax = b$ where A is an m by n matrix, x is a n-vector $\{x_i\}$ and b is an m vector of constant values $\{b_i\}$. (All entries are in GF_2 and operations are in GF_2.)

   (a) Use the method of conditional probabilities to show how to derive a deterministic algorithm that will find a solution for the x_i variables so as to yield a constant approximation on the number of satisfied equations. State the approximation ratio and argue why you obtain that ratio.

   (b) State the determinstic algorithm in terms that do not mention randomization in the same way that Johnson's algorithm is expressed (and was originally derived) without any reference to randomization.

5. Consider a constraint satisfaction problem CSP where each constraint is a Boolean function of at most k variables coming from a domain of n ternary variables (i.e. each variable can take on three values). Show how to modify Schoening's randomized algorithm so as to decide "with high probability" whether or not the CSP has a solution. Brute force search would result in a 3^n time method. The goal is to achieve a bounded error algorithm with time bound c^n for any $c < 3$.