CSC 2420S 2012, Assignment 1
Due: February 13 at start of class

1. Consider the jump local search algorithm for the makespan problem. On m identical machine this algorithm achieves an approximation ratio (locality gap) of $2 - \frac{2}{m+1}$.

   (a) Show that this bound is tight for identical machines; that is, find an input set S of jobs and an assignment A of jobs to machines such that A is locally optimal but has makespan 2 - 2/(m + 1) times the optimal makespan for S.

   (b) Show that this algorithm does not achieve any constant approximation for the makespan problem in the unrelated machines model.

2. Consider the makespan problem for the restricted machines model. Suppose we had an efficient algorithm ALG (say running in time $O(n^2)$) such that given any set of n jobs, ALG will output the value of an optimal makespan assignment. Show how to use such an algorithm to determine (in polynomial time) an assignment of the jobs so as to realize the optimal makespan value.

3. Consider the following *weighted partial vertex cover* problem: We are given a graph $G = (V, E)$ with node weights $c : V \to \mathbb{Q}^+$ and edge weights $d : E \to \mathbb{Q}^+$.‘ The goal is to find a partial cover $V' \subseteq V$ so as to minimize the total cost of nodes in the cover V' plus the cost of edges not adjacent to at least one node in V'; that is, to minimize $\sum_{v \in V'} c(v) + \sum_{e=(u,v):u,v \notin V'} d(e)$.

   (a) Provide a {0,1} IP for this problem.

   (b) Using an LP relaxation and rounding of the IP, what is the approximation ratio that you obtain?

4. Fill in the details for the proof of the interval selection approximation ratio of $Greedy_a$ as given on slide 15 of the lecture notes.