CSC 2420 Fall 2022, Assignment 3
Due date: Wednesday, December 7 at 1PM,)

It is certainly preferable for you to solve the questions without consulting a published source. However, if you are using a published source then you must specify the source and you should try to improve upon the presentation of the result.

If you would like to discuss any questions with someone else that is fine BUT at the end of any collaboration you must spend at least one hour playing video games or watching a Maple Leaf or Raptors game or maybe even start reading a good novel before writing anything down.

If you do not know how to answer a question, state "I do not know how to answer this (sub) question" and you will receive 20% (e.g. 2 of 10 points) for doing so. You can receive partial credit for any reasonable attempt to answer a question BUT no credit for arguments that make no sense.

1. Consider $n \times n$ matrices with entries from some infinite ring R (e.g., the integers). Someone claims to have an algorithm for multiplying two matrices A and B in time $O(n^{2.2})$ ring operations (i.e. $+, -, *$) assuming each such operation takes one time step. You are skeptical and don't see a proof but as long as you can use the algorithm and feel it works for your input matrices, you are happy. If the algorithm outputs the matrix C (i.e. C is supposed to be $A \cdot B$), you want to quickly vertify that the answer is correct with "high" probability. More specifically, if $C = A \cdot B$, your verifier will always say "correct" but if $C \neq A \cdot B$, your verifier will say "incorrect" with some desired probability p (say $p \geq .99999$). In order for the verifier to be useful, it must be much faster (say time $O(n^2)$) than the multiplcation algorithm. Describe such a verifier and sketch a proof as to why you are obtaining the desired probability.

   For those not familiar with the definition of a ring, please see the definition in Wikipedia.

2. Using the method of conditional expectations, describe what information is needed so that the naive randomized algorithm for the max-cut problem can be de-randomized into a deterministic $\frac{1}{2}$ competitive online algorithm. Having done that, can you describe the resulting algorithm without reference to the randomized algorithm?

3. Consider Schöning's random walk algorithm for 3SAT.

   - Consider a single trial. Note that the choice of unsatisfied clause in the algorithm is arbitrary. What would be a simple way to choose such a clause? Suggest some reasonable heuristics (to the choice of the unsatisfied clause and/or the varaiable(s) to flip) in the algorithm that might improve the probability of success "in practice" in a single trial. Briefly explain you reasoning for your proposed heuristics.
     Note: We are not looking for a provable improvement. We usually use the term *heuristic* to mean that we are suggesting modifications in an algorithm that we believe work well "in practice" but not something for which we have provable results.
   - Given the analysis on slide 30 of the L9 slides, what approximately would the probability of success be for a single trial if we did the random walk for 6n steps?

4. **Bonus:** Once again only do the bonus question if you have time and it will not interfere with other courses or your research. Create a random collection $\mathcal{C}$ of satisfiable formulas. State how you are generating random satisfiable formulas. Compare your heuristics for Schöning's algorithm in comparison to an implementation that does not use your heuristics or compare against the deterministic $O(2^{2-\delta}))$ algorithms. In particular, having chosen some number t of trials, how often do you fail to find a satisfying assignment?