CSC 2420 Fall 2022, Assignment 2
Due date: Wednesday, November 23 at 1PM,)

It is certainly preferable for you to solve the questions without consulting a published source. However, if you are using a published source then you must specify the source and you should try to improve upon the presentation of the result.

If you would like to discuss any questions with someone else that is fine BUT at the end of any collaboration you must spend at least one hour playing video games or watching a Maple Leaf or Raptors game or maybe even start reading a good novel before writing anything down.

If you do not know how to answer a question, state "I do not know how to answer this (sub) question" and you will receive 20% (e.g. 2 of 10 points) for doing so. You can receive partial credit for any reasonable attempt to answer a question BUT no credit for arguments that make no sense.

1. Fill in the details for the proof that the non-obivious local search algorithm for max-2-sat obtains a 3/4 approximation ratio. In particular, verify the "key lemma" stated on slide 4.

2. Consider again the weighted d-set packing problem that we saw in terms of greedy algorithms. Show that the following oblivious local search algorithm obtains a $\frac{1}{d}$ approximation and that this bound is tight for this algorithm:

   Choose any initial feasible solution G
   For any input set S_i, let $G_S = \{S_j \in G : S_i \cap S_j \neq \emptyset\}$
   WHILE there exists a set a set S such that $W(G \cup \{S_i\} \setminus G_S) > W(G)$

   $G = G \setminus G_S \cup \{S_i\})$
   END WHILE

3. This is a "thought question" regarding what I think is an open research problem. You will receive full credit for any "thoughtful" answer. For the uncapacitated facility location problem (UFL) or the k-metric median probelm (as defined in the slides for Week 7), discuss:

   - What you think might be a good way to define a non-oblivious local search algorithms.
   - How would you define a greedy algorithm for obtaining an initial solution.

     Note: You may want to look at a 2001 paper by Adam Meyerson (posted on web page) on how he defines an online and ROM algorithm for faciltiy location. Here the clients arrive online and the facitilies and their opening costs are known in advance. This then becomes a many to one bipartite matching problem. (He also considers the model where every client (city,location) can be a facility and where the opening cost is uniform.) Mettu and Plaxton also consider an online version of the k-median and facility location problems where any client (city, location) can be a facility and each client has a weight. I prefer thinking of F (the set of facilties) and C (the set of clients) as being distinct (although not necessarily disjoint) sets. Mettu and Plaxton use a random order algorithm as a initial solution to the local search algorithm of Arya et al. in order to improve the running time. Both papers give motivation for the facility location and k-median problems.
   - Bonus for those who have the time and the interest: Implement your algorithms. Is the non-oblivious algorithm better than the oblivious algorithm on (say) random instances? Does a greedy initialization make a significant difference?