CSC 2420 Fall 2022, Assignment 1
Due date: Wednesday, October 19 at 1PM,)

It is certainly preferable for you to solve the questions without consulting a published source. However, if you are using a published source then you must specify the source and you should try to improve upon the presentation of the result.

If you would like to discuss any questions with someone else that is fine BUT at the end of any collaboration you must spend at least one hour playing video games or watching a Blue Jays game or maybe even start reading a good novel before writing anything down.

If you do not know how to answer a question, state "I do not know how to answer this (sub) question" and you will receive 20% (e.g. 2 of 10 points) for doing so. You can receive partial credit for any reasonable attempt to answer a question BUT no credit for arguments that make no sense.

In class or on Piazza, I can clarify any questions you may have about this assignment or any material in the course. .

1. This question concerns the makespan problem on m identical machines.

   - [10 points]
     Argue for $m = 2$ (resp. $m = 3$) machines that *any* (not necessarily greedy) deterministic online algorithm would have competitive ratio no better than $\frac{3}{2}$ (resp. $\frac{5}{3}$) so that the natural greedy online algorithm approximation is tight for $m = 2$ and $m = 3$ for any online algorithm.
   - [5 points]
     Consider the nemesis sequence (i.e. $p_i = 1$ for $1 \leq i \leq m(m-1)$ and $p_{m(m-1)+1} = m$) that forces the ratio $2 - \frac{1}{m}$ for Graham's greedy algorithm. Consider the same sequence for the makespan problem on m machines but now in the random order model (ROM). Provide a "good" estimate for the expected competitive ratio of Graham's greedy algorithm iwhen executed on this set of inputs? Note: the expectation is wrt to the randomness in choosing the sequence uniformly at random. Your argument can just give good intuition but of course a proof is better.

Hint: The ROM model is equivalent to randomly choosing a time $y_i \in [0, 1]$ for the i^{th} item and then the ordering on the input jobs is defined by the ordering of the y_i variables. What is the expected location of the "big" item (i.e. $p_j = m$) in the random ordering?

- [5 points]

  Can you modify the above sequence to force a $2 - \epsilon$ lower bound on the expected competitive ratio for Graham's greedy algorithm in the ROM model ?

2. The following question involves the {0,1} knapsack problem. See slide 16 of week 2 slides for notation.

   - Show that the following greedy algorithms cannot achieve a constant approximation ratio:
     1) [5 points] sort items so that $s_1 \leq s_2 \ldots \leq s_n$ and accept items greedily (i.e. if the item fits place it into the knapsack).
     2) [5 points] sort items so that $v_1 \geq v_2 \ldots \geq v_n$ and accept items greedily.
     3) [5 points] sort items so that $v_1/s_1 \geq v_2/s_2 \ldots \geq v_n/s_n$ and accept greedily.

   - [10 points]

     Show that with one bit of randomness that there is a greedy (i.e. priority) algorithm that achieves (in expectation) a $\frac{1}{2}$ approximation ratio.

   - [5 points]

     Show that no deterministic priority algorithm can achieve a constant approximation ratio. **Note:** This question may be the hardest one on this assignment. Do nort spend too much time on this or any question.