1. Let L be an $\mathcal{NP}$ complete language and let R be a polynomial time verification algorithm for L with certificate polynomial bound $p(n)$. That is,

$$x \in L \text{ iff } \exists y[R(x, y = 1 \ \text{ and } |y| \leq p(|x|)]$$

Show that the "functional" problem $f_R(x)$ is polynomial time reducible to the decision problem for L.

2. (a) Let $L = \{C|\ C$ is a circuit having n inputs and size $\leq n^{\log_2 n}$ and C is the lexicographic first circuit for the function computed by C $\}$. Show that L is computable in (uniform) space $n^{2\log_2 n}$.

   (b) Show that there is a language which is computable in (uniform) space $n^{\log_2 n}$ but not in $\mathcal{POLYSIZE}$.