CSC2401F **Problem Set 1** **Due: Oct 2,1997**

1. Consider the language of palindromes $L = \{xx^R | x \in \{0,1\}^*\}$. Let $\mathcal{M}$ be a multi-tape T.M. which decides L. Suppose that M is space bounded by S and time bounded by T. Show $T \cdot S = \Omega(n^2)$.

   Hint: Adapt the proof that L requires time $\Omega(n^2)$ on a single tape T.M.

2. Let $\mathcal{M}$ be a multi-tape non deterministic T.M. operating within time bound T. Show that there is an equivalent non deterministic machine $\mathcal{M}'$ with 2 work tapes which operates in time $O(T)$.

   Hint: First assume $\mathcal{M}$ is deterministic. To simulate $\mathcal{M}$, $\mathcal{M}'$ first guesses for each time step $t \leq T$ the state of $\mathcal{M}$ and the symbol being read on each tape. Then $\mathcal{M}'$ must verify that indeed these guesses were correct.

3. Suppose functions f and g are both computable in deterministic space S(n) and that the length of $g(x)$ is polynomial in the length of x for every input x. Show $f \circ g$ is also computable in deterministic space $S(n)$.

4. Suppose that $NTIME(n) \subset DTIME(n^k)$ for some k. Show $\mathcal{P} = \mathcal{NP}$