CSC2401F Instructor: A. Borodin
Text: "Theory of Computational Theory: A Modern Approach" by Sanjeev Arora and Boaz Barak

CSC2401 is an introductory level graduate course which is appropriate for all graduate students in Computer Science, as either a breadth course for those not primarily interested in theoretical aspects of computing or as a foundational course for those students who may be planning to focus on theoretical issues. The first and main part of the course will be an introduction to complexity theory where we discuss uniform and non-uniform models of computation, time and space complexity classes, complexity hierarchies, reductions and completeness, randomization in time and space computations, tradeoff issues and provably hard problems. In the latter part of the course we will discuss (time permitting) some relatively new results in complexity theory such as time-space lower bounds, derandomization results, PCP proofs and inapproximation results. Students can access previous versions of this course as perhaps the best indicator of what we intend to be doing. See www.cs.toronto.edu/~bor/2401f01, 2401f02, 2401f03. and www.cs.toronto.edu/~rackoff/2401f00.

Here follows a very tentative (and overly ambitious) schedule of topics.

1. The multitape Turing machine model; time and space complexity measures and complexity classes. Why the computational model may not matter.

2. The polynomial time complexity thesis.

3. The basic diagonalization technique; time (space) complexity class hierarchies.

4. Non determinstic computation.

5. The basic inclusions :

$$NC^1 \subseteq L \subseteq NL \subseteq P \subseteq NP \subseteq PSPACE \subseteq \bigcup_k DTIME(2^{n^k}) = EXP$$

6. Logspace reducibility; $STCON$ (directed graph s-t connectivity) is complete for NL wrt logspace reducibility.
   $NSPACE(s(n)) \subseteq DSPACE(s(n)^2)$. The relation between space and parallel time.

7. $\overline{STCON} \in NL$ and hence $NL = coNL$.

8. BHP (bounded halting problem), $CIRCUIT-SAT$, SAT, etc. etc. are complete for NP.

9. Fixed Parameter Tractable Problems

10. The polynomial hierarchy; alternative characterizations of NP and Σ_k^p.

11. Boolean circuit complexity and non-uniform complexity measures of size and depth. Why are circuit lower bounds so difficult. Algebraic cirucit complexity.

12. Alternating Turing machines.
    $ATIME(logn) = uniformNC^1$ (log parallel time); $ASPACE(logn) = P$.

13. Provably hard natural languages.
    TRE = totality of regular expressions (resp. $TERE$ = totality of extended regular expresssions) is complete for polynomial (resp, exponential) space.

14. Indirect diagonalization and time-space bounds.
    $SAT \notin DTISP(n^{\sqrt{3}}, n^{o(1)})$.

15. Randomized computation; symbolic determinant; primality testing in P.
    $BPP \subseteq POLYSIZE$ and $BPP \subseteq \Sigma_2^p$. $P = BPP$ if $P = NP$.

16. $\#P$ (counting problems) and IP (interactve proofs). $\#P$ in IP; $IP = PSPACE$.

17. Pseudo random generators. $P = BPP$ if there exists $L \in E$ such that L has circuit complexity $2^{\Omega(n)}$.

18. $USTCON$ (undirected graph s-t connectivity) in L; $L = SL$.