CSC 2401F 2007, Assignment 2
Due: November 22, 1:10 PM

1. Let $G = (V, E)$ be a directed graph and let $ODDSTCON = \{(G, u, v) | \exists$ a path from u to v and all paths from u to v have odd length$\}$.
   Show that $ODDSTCON$ is complete for $\mathcal{NL}$ wrt $\leq_{log-space}$ by showing:

   (a) $ODDSTCON$ is in $\mathcal{NL}$.

   10 points

   (b) $ODDSTCON$ is hard for $\mathcal{NL}$ wrt $\leq_{log-space}$.

   10 points

2. Do one of the following

   - Consider regular expressions with a "concatenation squaring operator " R^2 such that $L[(R)^2] = L[R] \circ L[R]$. Call such expressions Sregular expressions. Show that $\{R | R$ is a Sregular expression over Σ such that $L[R] \neq \Sigma^*\}$ is hard for exponential space.

     10 points

   - Consider extended regular expressions which are regular expressions with the addition of an intersection operator. Show that $\{R | R$ is an extended regular expression over Σ such that $L[R] \neq \Sigma^*\}$ is hard for exponential space. 20 points

3. (a) Show that for every k, there is a language L in PH such that $L \notin SIZE(O(n^k))$.
   Hint: by a counting argument show that there exists a language L such that for sufficiently large n, L is in $SIZE(n^{k+2})$ but not in $SIZE(n^{k+1})$ and hence not in $SIZE(O(n^k)$.

   20 points

   (b) Using the first part of this question, show that $P \subseteq SIZE(O(n^k)$ for some fixed k implies $P \neq NP$.

   10 points

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