CSC 2401F 2007, Assignment 1
Due: Oct. 18, 1:10 PM

1. (10 points) Let's define a more "honest" measure of T.M. space complexity for which the space compression result does not hold. Suppose we consider a deterministic Turing machines with a read-only binary input tape and one work tape with work tape alphabet Γ.
   If a T.M. $\mathcal{M}$ has q states and visits s worktape squares on input w then we will say that $\mathcal{M}$ uses $HSPACE\ s \log_2 |\Gamma| + \log_2 |w| + \log_2 s + \log_2 q$.

   Show that there exists a constant c such that there is a decision problem in $HSPACE(c \cdot n) - HSPACE(n)$.

2. (10 points) Suppose $NL \subseteq DSPACE(\log^{1.5})$. Show that for every space constructible $S(n) \geq \log n$ that $NSPACE(S) \subseteq DSPACE(S^{1.5})$.
   Hint: see Theorem 2.25

3. (a) (10 points) Exercise 13 of Chapter 2.
   (b) (10 points) Exercise 23 of Chapter 2. Note that this result and the previous result can be viewed as an alternative to Ladner's result which states that "$P \neq NP$ implies there exists a non complete $L \in NP - P$". We now get the same conclusion of a (unary) language $L \in NP - P$ but now we need the stronger assumption that $EXP \neq NEXP$.

4. (10 points) Exercise 2 of chapter 3.
   Hint: Consider a language L and its polynomial padded language $L^{pad} = \{< w, 1^{n^2} > | w \in L\}$.