CSC 2401F 2003, Assignment 3
Due: Dec. 19, 4PM

1. Assume that the polynomial hierarchy does not collapase. That is, $\Sigma_k^P \subset \Sigma_{k+1}^P$ for all $k \geq 0$ where $\subset$ denotes proper inclusion. Prove or disprove the following: If $L \in NP^B$ and $B \in NP^C$ then $L \in NP^C$.

   Hint: This is mainly a definitional question and can be viewed as a Christmas gift.

2. Do exactly one of the following two questions:

   - Show that E^{PH} contains a language L such that $\forall n[L|_n$ requires $MAXC(n)$ circuit size]. Conclude that
     (a) $ESPACE$ contains a language L such that $\forall n[L|_n$ requires $MAXC(n)$ circuit size].
     (b) If $\forall L \in E\, \exists n[L|_n$ can be computed with at most $MAXC(n)-1$ gates] then $P \neq NP$.

     Hint: This is similar to the proof that if $P \in SIZE(O(n^k)$ for any fixed k then $P \neq NP$.

   - Show that if $PSPACE \subseteq POLYSIZE$ then $PSPACE = \Sigma_2^P$.
     Hint: This is similar to the proof that if $NP \in POLYSIZE$ then $PH = \Sigma_2^P$. In particular, you should prove that the statement L(C) = "Circuit C computes QBF" is a Π_1 statement. Also you can use the fact that QBF is complete for PSPACE.

3. Consider an extension of regular expressions (which we will call squegular exprssions) which in addition to the usual language concatenation, union and Kleene closure operations, also has a "squaring" operation R^S whose semantics is defined by $L[R^S] = L[R] \circ L[R]$. Consider the language $\overline{TSRE} = \{R | R$ is a squegular expression and $L[R] \neq \Sigma^*.\}$. Show that $\overline{TSRE}$ is complete for ESPACE (say wrt linear space transformations).

   Hint: Just show how to modify the argument that $\overline{TRE}$ is complete for PSPACE.