1. Assume that the polynomial hierarchy does not collapse. That is, $\Sigma_k^P \subset \Sigma_{k+1}^P$ for all $k \geq 0$ where $\subset$ denotes proper inclusion. Prove or disprove the following: If $L \in NP^B$ and $B \in NP^C$ then $L \in NP^C$.

Hint: This is mainly a definitional question and can be viewed as a Christmas gift.

2. Do exactly one of the following two questions:

- Show that $E^{PH}$ contains a language $L$ such that $\forall n |L|_n$ requires $MAXC(n)$ circuit size. Conclude that
  (a) $ESPACE$ contains a language $L$ such that $\forall n |L|_n$ requires $MAXC(n)$ circuit size.
  (b) If $\forall L \in E \exists n |L|_n$ can be computed with at most $MAXC(n) - 1$ gates then $P \neq NP$.

Hint: This is similar to the proof that if $P \in SIZE(O(n^k))$ for any fixed $k$ then $P \neq NP$.

- Show that if $PSPACE \subset POLYSIZE$ then $PSPACE = \Sigma_2^P$.

Hint: This is similar to the proof that if $NP \subset POLYSIZE$ then $PH = \Sigma_2^P$. In particular, you should prove that the statement $L(C) = "\text{Circuit C computes QBF}"$ is a $\Pi_1$ statement. Also you can use the fact that QBF is complete for PSPACE.

3. Consider an extension of regular expressions (which we will call s排队ular expressions) which in addition to the usual language concatenation, union and Kleene closure operations, also has a "squaring" operation $R^S$ whose semantics is defined by $L[R^S] = L[R] \circ L[R]$. Consider the language $TRE = \{R | R \text{ is a s排队ular expression and } L[R] \neq \Sigma^* \}$. Show that $TRE$ is complete for ESPACE (say wrt linear space transformations).

Hint: Just show how to modify the argument that $TRE$ is complete for PSPACE.