CSC 2401F 2003, Assignment 2

Due: Nov. 13, 10:10 AM

1. Let $A \subseteq \Sigma^*$ be NP-complete.
   (a) If $B \in P$, $A \cap B = \emptyset$, and $A \cup B \neq \Sigma^*$. Prove that $A \cup B$ is NP-complete with respect to $\leq_{polytime}$..
   (b) Is the condition $A \cap B = \emptyset$ necessary in part (a).
2. Consider the graph colouring optimization problem GCO and its natural decision problem GCD.
   (a) Show that $GCO \leq^T_{polytime} GCD$.
   (b) Given that it is NP-hard to decide if a graph is 3-colourable, show that it is NP-hard to obtain a polynomial time c-approximation algorithm for any constant c.

      Note: It is actually NP-hard to obtain a polynomial time $n^{1-\epsilon}$-approximation for any $\epsilon > 0$ but that is a difficult result using the PCP framework. The problem here can be solved using some relatively elementary graph theoretic ideas.
3. Consider the class BPP^A, the class of all languages accepted by a polynomial time bounded probabilistic TM with oracle A and with error probability $\leq 1/4$. Suppose $A \in BPP$. Show that $BPP^A = BPP$.
4. More to follow.