CSC 2401F 2003, Assignment 1
Due: Oct. 16, 10:10 AM

1. Prove that there is a language L decided by a $k+1$-tape TM running in time T_2 but not decided by any k-tape TM running in time T_1 whenever $\lim_{n\to\infty} \frac{T_1(n)}{T_2(n)} = 0$.

2. Use a padding argument to show that $DTIME(n)$ is properly contained in $DTIME(n \log n)$. Is $DTIME(n)$ properly contained in $DTIME(n\sqrt{\log n})$?

3. Complete the proof that $2-SAT$ is in NL.

   Namely, show that if the graph G_F does not have a "bad node x", then F is staisfiable. A node x is bad if in G_F there is a directed path from x to $\bar{x}$ and a directed path from $\bar{x}$ to x.
   Hint: for any nodes u and v, if there is a directed path from u to v, then there is also a directed path from $\bar{v}$ to $\bar{u}$.

4. Prove that $2-SAT$ is hard for NL wrt logspace transformations; that is, show for any $L' \in NL$ that $L' \leq_{logspace} 2-SAT$.