Computer Science 2401F — October 1, 2002
St. George Campus — University of Toronto

Homework Assignment #1
Due: October 22, 2002, by 1:10PM

**Question 1.** (10 marks) This question gives a particular provable example of a hierarchy theorem about *simultaneous* time and space.

Define DTIME-DSPACE$(t(n), s(n))$ to consist of those languages $L$ that are accepted by some Turing machine that runs in time $t$ and also in space $s$. Prove that there is a language contained in DTIME-DSPACE$(n^4, n\sqrt{n})$ − DTIME-DSPACE$(n^3, n)$.

**Question 2.** (10 marks) It turns out that if we use a more realistic space measure for Turing machines, then the constant factor space-compression theorem goes away, allowing us to prove a stronger hierarchy theorem. We will consider here only Turing machines with a single worktape and a binary input alphabet.

Let $M$ be a Turing machine that on an input $x$ of length $n$ visits $s$ worktape squares. A realistic measure of space is to count $s \log |\Gamma|$ for the worktape space (where $\Gamma$ is the set of worktape symbols), plus $\log n$ for the (implicit) storage of the input head position, plus $\log s$ for the (implicit) storage of the worktape head position, plus $\log |Q|$ for storing the state (where $Q$ is the set of states). So we say the RealSpace used is $s \log |\Gamma| + \log n + \log s + \log |Q|$.

Exhibit a constant $c$ and prove there is a language contained in RealSpace$(c \cdot n)$ − RealSpace$(n)$.

**Question 3.** (10 marks) Assume for this question (although it is unlikely) that there is a constant $k$ such that NTIME$(n) \subseteq$ DTIME$(n^k)$. Prove that **P=NP**.

**Question 4.** (10 marks) The point of this question is to show that unlike the case with deterministic machines, a nondeterministic machine with only 2 worktapes can efficiently simulate a machine with many tapes.

Let $M$ be a nondeterministic Turing machine such that on every input of length $n$ that is accepted, there is some computation that halts with $t(n)$ steps.

Describe how to construct a nondeterministic Turing machine $M'$ that has only 2 worktapes, that accepts the same language as $M$, and such that on every input of length $n$ that is accepted, there is some computation that halts within $O(t(n))$ steps.

Hint: Use one of the tapes to guess some information about an accepting computation.