CSC 2401 Introduction to Complexity – Fall 2001
Lecture #8, Tuesday, October 09, 2001
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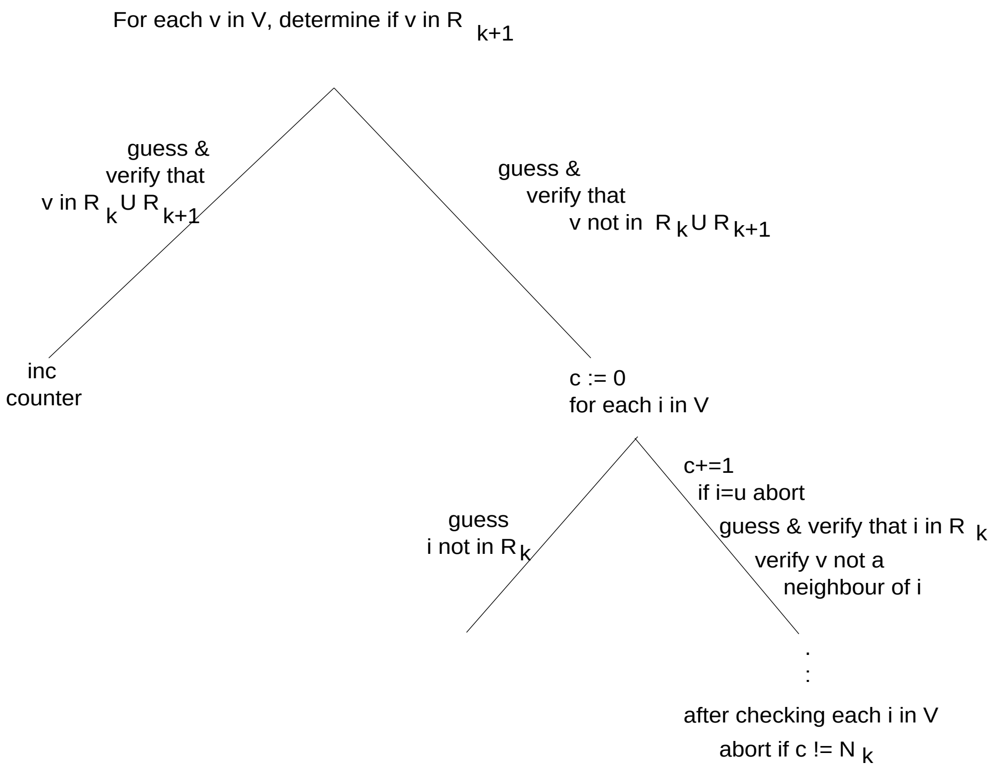
8.1 Immerman and Szelepcsényi's result

Recall from the last lecture that to prove $coNL = NL$, it is sufficient to show $\overline{STCON} \in NL$, where $\overline{STCON} = \{(G, u, v) | \nexists \text{ path from } u \text{ to } v\}$. The method is twofold:

1. Compute N_k (inductively on k) in non-deterministic logspace. We define $R_k = \{v | v \text{ is reachable from } u \text{ by a path of length} \leq k\}$ and $N_k = |R_k|$
2. $(G, u, v) \in \overline{STCON}$ iff $v \in R_{n-1}$. Given N_{n-1} show how to determine in non deterministic log space if $v \in R_{n-1}$.

Proof: $N_0 = 1$. Inductive step:
Suppose we know N_k. Show how to compute N_{k+1} (in non-deterministic logspace).

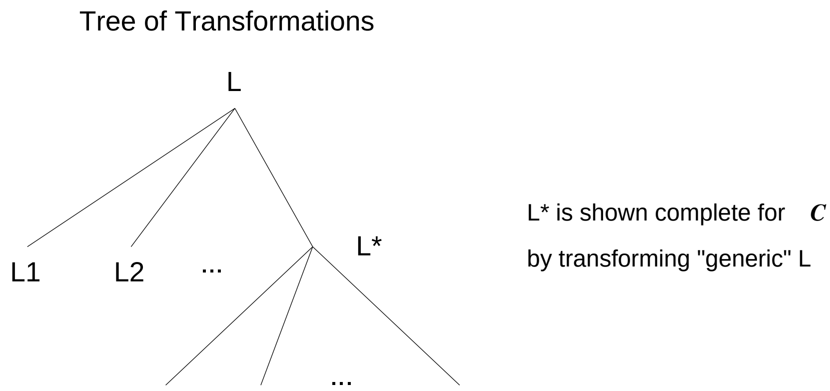


8.2 NP-Completeness

To show that some language L is complete for $\mathcal{C}$ wrt $\leq_R$

1. Show $L \in \mathcal{C}$.
2. (a) Take a "generic" $L' \in \mathcal{C}$ and show $L' \leq_R L$.
   OR

   (b) Knowing $\tilde{L}$ is complete for $\mathcal{C}$ wrt $\leq_R$, show $\tilde{L} \leq_R L$. (By transitivity, it follows that L is hard for $\mathcal{C}$).



Definition 1 *$NP = \{L | L$ is accepted by some non-deterministic TM running in time $p(n)$, p is polynomial.$\}$*

Fact: If $L \in NP$ then

- L is accepted in time $\tilde{p}(n)$ (by some non-deterministic TM M) where $\tilde{p}$ is a polynomial which is logspace constructible.
- $\forall w$, M halts on w within $\tilde{p}(|w|)$ steps.

Definition 2 *L is complete for NP wrt $\leq_{logspace}$ or wrt $\leq_p$ (AKA NP-complete; when stated this way, one usually assumes that the transformation is $\leq_p$) if*

- *$L \in NP$*
- *L is hard for NP wrt $\leq_{logspace}$ (or wrt $\leq_p$)*
  that is, $\forall L' \in NP$, $L' \leq_{logspace} L$.

(State of the art: $w \in L'$ iff $f(w) \in L$, wave hands to show that f is efficiently computable.)

8.3 Cook's Theorem

Theorem 1 *SAT (CNF-SAT, 3CNF-SAT) are NP-complete.*

Other problems such as CLIQUE are also NP-complete. PRIMES $\in NP$ but apparently not complete.

Theorem 2 *The Bounded Halting Problem (BHP) is NP-complete, where BHP* $= \{< x, y, 0^t > \mid$ *non-deterministic* M_x *accepts* y *using* $\leq t$ *steps*$\}$.

We need to show

1. BHP $\in NP$ (can be proved by simulation).
2. Let $L = L(M)$ such that M always halts in $p(n)$ steps, p constructible. L $\leq_{logspace}$ BHP via f where $y \mapsto_f < x, y, 0^{p(|y|)} >$.

Preview: Show CIRCUIT-SAT is NP-complete, where
CIRCUIT-SAT = { $\bar{C}|\bar{C}$ encodes a boolean circuit C in some number of inputs $y_1, \ldots, y_m$ and $\exists$ a setting of these inputs $y_i = a_i \in \{0, 1\}$ for which C outputs 1}