CSC2401F **Problem Set 2** **Due: Nov 8, 2001**

1. Let $G = (V, E)$ be a directed graph and let
   $ODDSTCON = \{(G, u, v) | \exists$ a path from u to v and all paths from u to v have odd length$\}$.
   $Show that$ODDSTCON is complete for $\mathcal{NL}$ wrt $\leq_{log-space}$ by showing:

   (a) $ODDSTCON$ is in $\mathcal{NL}$.

   (b) $ODDSTCON$ is hard for $\mathcal{NL}$ wrt $\leq_{log-space}$.

2. The goal of this problem is to give some evidence that integer factoring is not complete. First we have to define an appropriate language version of integer factorization.
   Recall $PRIME = \{x | x$ x is a binary encoding of a positive prime integer $(x)_2$ $\}$
   It is a non trivial fact that $PRIME$ is in NP.
   We now define $FACTOR = \{(x, m) | x$ and m are binary encodings of positive integers $(x)_2$ and $(m)_2$ such that $(x)_2$ is a composite number which has a non-trivial factor less than $(m)_2$ $\}$.

   Using the fact that $PRIME$ is in NP show that :

   (a) $FACTOR$ is in NP.

   (b) The complement of $FACTOR$ is also in NP.

3. Garey and Johnson define problems called 3-PARTITION and 4-PARTITION. The definition is not that intuitive since the standard PARTITION problem is NOT the same as what they would call 2-PARTITION.
   Definition: k-PARTITION $= \{(a_1, \ldots, a_{km} | \exists$ subsets of indices $S_1, \ldots, S_m$ where $\forall j$ $|S_j| = k$ and $(\sum_{i \in S_j} a_i) = (\sum_{1 \leq i \leq km} a_i)/m$ $\}$.

   (a) Show that 2-PARTITION is in P.

   (b) Garey and Johnson provide a proof showing that 3-PARTITION is NP-hard in the strong sense. Use this fact to show that the following scheduling problem is NP-hard in the strong sense:

   Given n jobs $J_1, \ldots, J_n$ where job $J_i = (r_i, d_i, p_i, w_i)$. This is the usual scheduling notation where r_i is the release time, d_i is the deadline, p_i is the processing time and w_i is the weight or reward of job i. The optimization problem is to derive as much profit as possible from a feasible schedule (on one machine) of some subset of the jobs where a scheduled job must begin no sooner than the release time and complete no later than the deadline and in a feasible schedule no two scheduled jobs are running at the same time.