CSC 238S **Term Test #2** **March 18/96**

Name ______________________ Student No. ______________________

Tutorial SF1101 ______________ GB120 ______________ GB 305 ______________

Open Book
Duration - 1 hr. 20 min.
Answer ALL questions on test paper. Use backs of sheets for scratch work.
Total Marks: 50

[15] 1. Express the following Boolean functions in CNF (Conjunctive Normal Form).
(As usual, $\oplus$ denotes Exclusive OR and $\neg$ denotes Negation.)

i) $x_1 \oplus x_2$
Hint: $\neg(x_1 \oplus x_2) \equiv \neg x_1 \oplus x_2$

ii) $f(x_1, x_2, x_3, x_4) = \begin{cases} \text{TRUE} & \text{if } x_1 = x_2 \text{ and } x_3 = x_4 \\ \text{FALSE} & \text{otherwise} \end{cases}$
Hint: $\neg(x_1 \oplus x_2) \equiv$ "$x_1 = x_2$"

iii) $f(x_1, x_2, x_3, x_4) = \begin{cases} \text{TRUE} & \text{if } x_1 = x_2 = x_3 = x_4 \\ \text{FALSE} & \text{otherwise} \end{cases}$
Hint: You should be able to do this with $6 = 2 \cdot 3$ clauses.

[10] 2. Consider the following interpretation. Assume that the Domain is the set of non negative integers, and that the following symbols exist and are interpreted as follows.

predicates:	EQ(X, Y)	signifies	"$X = Y$"
	GE(X, Y)	signifies	"$X \geq Y$"
	MULT(X, Y, Z)	signifies	"$X \times Y = Z$"
constants:	0,1,2		

Write an expression in predicate logic to represent each of the following:

(a) X is a prime number (i.e. write a formula with one free variable X).

(b) There are arbitrarily large prime numbers (i.e. write a formula with no free variables representing this fact). You may use PRIME(X) to denote "X is prime".

3. Assume we have a set of individuals as a Domain, and that the following symbols exist and are interpreted as follows.

predicates:	PARENT(X, Y)	signifies	"X is a biological parent of Y"
	M(X)	signifies	"X is a male"
	F(X)	signifies	"X is a female"

[9] (a) Express (in English) each of the following "axioms":

i) F_1 $\forall X\, ((\mathrm{M}\ (X) \vee \mathrm{F}\ (X)) \wedge (\mathrm{M}(X) \to \neg \mathrm{F}(X))$

ii) F_2 $\forall Y \exists X_1 \exists X_2\, (\mathrm{PARENT}\ (X_1, Y) \wedge \mathrm{PARENT}\ (X_2, Y) \wedge \mathrm{M}\ (X_1) \wedge \mathrm{F}\ (X_2))$

iii) F_3 $\forall X_1 \forall X_2 \forall Y\, ((\mathrm{PARENT}\ (X_1, Y) \wedge \mathrm{PARENT}\ (X_2, Y) \wedge X_1 \neq X_2)$
$\to ((\mathrm{M}\ (X_1) \to \mathrm{F}\ (X_2)) \wedge (\mathrm{F}\ (X_1) \to \mathrm{M}\ (X_2))))$

[16] (b) Consider the following definitions:

GRANDPARENT (X, Y) :

$$\exists Z\,(\text{PARENT }(Z, Y) \wedge \text{PARENT }(X, Z))$$

COUSIN (X, Y):

$$\exists Z\,(\text{GRANDPARENT }(Z, X) \wedge \text{GRANDPARENT }(Z, Y))$$

Which of the following logically follows from the axioms F_1, F_2, F_3; i.e. in any interpretation making F_1, F_2 and F_3 true, the statement is true. Explain your answer briefly. Note that an interpretation need not correspond to "reality".

i) An individual has exactly 2 (different) parents

ii) An individual has exactly 4 (different) grandparents

iii) The COUSIN relation is symmetric; i.e.

$$\forall X \forall Y\,(\text{COUSIN }(X, Y) \rightarrow \text{COUSIN }(Y, X))$$

iv) No one is their own parent; i.e.

$$\forall X \neg\, \text{PARENT }(X, X)$$