Computer Science 238 | November 22, 1995
Erindale Campus | University of Toronto

Homework Assignment #5
Due: Wednesday, December 6, 1995 by 2:10pm

On the cover page of your assignment you must write **and sign** *the following statement: "I have read and understood the policy on collaboration in homework stated in the Course Information handout." Without such a signed statement your homework will not be marked.*

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1. (15 points) Construct deterministic FSA to accept each of the following languages over $\{0, 1\}$. In each case, prove that your FSA accepts the corresponding language.

(a) $\{x :$ each 0 in x is immediately preceded and followed by a 1$\}$.

(b) $\{x :$ neither 00 nor 11 appear as substrings of $x\}$.

(c) $\{x :$ both 00 and 11 appear as substrings of $x\}$.

2. (15 points) Let L be a regular language over Σ. Prove that each of the following sets is also regular:

(a) The set of prefixes of L, $\text{Prefix}(L) = \{x \in \Sigma^* : xy \in L, \text{for some } y \in \Sigma^*\}$.

(b) The set of suffixes of L, $\text{Suffix}(L) = \{x \in \Sigma^* : yx \in L, \text{for some } y \in \Sigma^*\}$.

(c) The set of maximal strings of L, $\text{Max}(L) = \{x \in L : \text{for any } y \in \Sigma^*, \text{ if } y \neq \epsilon \text{ then } xy \notin L\}$. (Hint: Show that $\text{Max}(L)$ is a finite language — even if L is infinite.)

3. (15 points) Prove or disprove whether each of the following equivalences holds for arbitrary regular expressions R and S.

(a) $(R^*S^*)^* \equiv (R|S)^*$.

(b) $(R|S)R^* \equiv (S|\epsilon)R^*$.

(c) $(RS|R)^*R \equiv R(SR|R)^*$

4. (10 points) A regular expression is called $\emptyset$-*free* if it does not contain, as a subexpression, the regular expression $\emptyset$. More precisely, the set of $\emptyset$-free regular expressions is defined inductively just as the set of regular expressions (see lecture notes, page 8) except that in the basis of the definition we omit the expression $\emptyset$. Prove that any regular expression is equivalent either to an $\emptyset$-free regular expression, or to the regular expression $\emptyset$. (Hint: Use structural induction on the definition of regular expressions and recall the following identities: $R|\emptyset \equiv \emptyset|R \equiv R$, $R\emptyset \equiv \emptyset R \equiv \emptyset$, and $\emptyset^* \equiv \epsilon$. You may use these identities without proving their validity.)

THAT'S IT WITH HOMEWORK, FOLKS