CSC238S **Problem Set 3** **Due: March 18/96**

1. a) Show $\{\rightarrow, \neg\}$ is functionally complete. You may use the fact that $\{\wedge, \vee, \neg\}$ is functionally complete, as well as any known propositional equivalences.

   b) Prove $\{\oplus, \neg\}$ is *not* functionally complete.
   Hint: $\neg(x_1 \oplus x_2 \oplus x_3 \oplus \cdots \oplus x_n) \equiv (\neg x_1 \oplus x_2 \oplus \cdots \oplus x_n)$.

   How many truth assignments $\tau : \{x_1, \cdots, x_n\} \rightarrow \{0, 1\}$ satisfy a formula which uses only $\oplus$ and $\neg$?

2. Problem 14.4.2 on p.722.

3. Transform each of the following formulas into a logically equivalent formula in Prenex Normal Form.

   (a) $(\forall x\, A(x) \vee \exists x\, B(x)) \rightarrow C(x)$.

   (b) $(\exists x\, A(x) \rightarrow \forall x\, B(x)) \rightarrow \exists x\, C(x)$.

4. Consider the following statements about relationships between people:

   (a) Everyone likes at least two people.

   (b) Nobody likes everybody.

   (c) Everyone likes themself but not everyone is liked by someone else.

   The table below lists the predicate symbols in a first-order language, along with their intended meanings:

Predicate Symbol	Intended Meaning
$l(X, Y)$	"X likes Y"
$n(X, Y)$	"$X \neq Y$"

   Translate each of the above statements to a predicate logic formula.

5. Consider the following closed formula F in predicate logic:

   $$\forall X\, \forall Y\, \exists Z\, (p(X, Y) \rightarrow p(X, Z) \wedge p(Z, Y))$$

   a) Find an interpretation for which F is true and an interpretation for which F is false.

   b) Informally argue (without reference to any particular interpretation) why $(F \rightarrow G)$ is valid where G is $\forall X\, \forall Y\, \exists Z_1\, \exists Z_2 (p(X, Y) \rightarrow p(X, Z_1) \wedge p(Z_1, Z_2) \wedge p(Z_2, Y))$.