Computer Science CSC-238F — Wednesday 25 October 1995
Erindale Campus — University of Toronto

Homework Assignment #3
Due: Wednesday 8 November 1995, 2:10 P.M.

1. [10 points] Transform each of the following formulas into a logically equivalent formula in Prenex Normal Form.

(a) $\forall x\, A(x) \to \exists x\, B(x)$.

(b) $\forall x\, A(x) \leftrightarrow \exists x\, B(x)$.

(c) $(\forall x\, A(x) \wedge \forall x\, B(x)) \to C(x)$.

(d) $(\exists x\, A(x) \to \exists x\, B(x)) \to \exists x\, C(x)$.

(e) $(\exists y\, D(x,y) \vee \forall x\, A(x)) \to \forall x\, \big((A(x) \wedge \exists u\, E(x,u)) \to (\exists y\, D(x,y) \to \exists x\, E(u,x))\big)$.

2. [10 points] Consider the following sentences in the predicate calculus:

(i) $\forall x\, \exists y\, (P(x,y) \wedge A(y) \wedge B(y))$

(ii) $\forall x\, \exists y\, P(x,y) \to \exists x\, \forall y\, P(x,y)$.

Define an interpretation $\mathcal{I}$ as follows: The domain D is the set of all natural numbers $\{0, 1, 2, \ldots\}$, $A(x)$ holds when x is divisible by 2, $B(x)$ holds when x is divisible by 3, and $P(x,y)$ holds when $x \leq y$. For each of the sentences above, do the following:

(a) Write down its meaning (using normal English) under interpretation $\mathcal{I}$.

(b) Say whether it is `true` or `false` under this interpretation (with a short explanation).

(c) Find another interpretation for which the sentence has the opposite truth-value (and explain why your interpretation works).

3. [10 points] Consider the following statements about relationships between people:

(a) Someone is unmarried.

(b) Nobody is married to a cousin (where cousins are defined as persons sharing at least one grandparent).

(c) Someone is a child of two unmarried persons.

(d) Not all married couples have children.

The table below lists the predicate symbols in a first-order language, along with their intended meanings:

Predicate Symbol	Intended Meaning
$\mathrm{P}(x, y)$	"x is a parent of y"
$\mathrm{M}(x, y)$	"x is married to y"

Translate each of the above statements to a predicate calculus formula.

NOTE: In this exercise, you must make some assumptions about the meaning of normally familiar terms—for instance, that "x is a child of y" is equivalent to "y is a parent of x". State all such assumptions!