Computer Science 238
 Erindale Campus

September 20, 1995
 University of Toronto

Homework Assignment #1
Due: Wednesday, October 4, 1995 by 2:10pm

On the cover page of your assignment, you must write **and sign** *the following statement: "I have read and understood the policy on collaboration in homework stated in the Course Information handout." Without such a signed statement your homework will not be marked.*

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1. (10 points) Nirvana Universal Trucking (NUT) provides transportation service between $n \geq 2$ cities, using trucks. Each NUT truck is dedicated to transporting goods back and forth between a specific pair of the n cities. Only some pairs of cities have NUT trucks running between them, but it is possible to ship goods from any one of the cities to any other — possibly going through some intermediate cities — using only NUT trucks. Prove that NUT has *at least* $n-1$ trucks.

2. (10 points) Which amounts of money can be formed by using only dimes and quarters? Prove your answer by using a form of mathematical induction.

3. (10 points) Let A be a set and $\sqsubseteq$ be a binary relation on A. We write $a \sqsubseteq b$ if a is related by $\sqsubseteq$ to b. The relation $\sqsubseteq$ is a *partial order on* A if it satisfies both of the following properties:

- (anti-symmetry) for any $a, b \in A$, if $a \sqsubseteq b$ and $b \sqsubseteq a$ then $a = b$.
- (transitivity) for any $a, b, c \in A$, if $a \sqsubseteq b$ and $b \sqsubseteq c$ then $a \sqsubseteq c$.

Anti-symmetry states that if a is related to b and b to a then a and b are, in fact, equal. Transitivity states that if a is related to b and b to c, then a is related to c.

Here are some examples of familiar relations that are partial orders: Both $\leq$ and $\geq$ are partial orders on $\mathbf{N}$. The relation "is (evenly) divisible by" is a partial order on $\mathbf{N}$. If X is an arbitrary set, then $\subseteq$ is a partial order on the powerset of X.

Let B be a subset of A. We say that an element b^* is a *minimal* element of B (with respect to partial order $\sqsubseteq$) if (a) $b^* \in B$, and (b) for all $b \in B$, if $b \sqsubseteq b^*$ then $b = b^*$.

i. (5 points) Using induction, prove that every nonempty finite subset of A has a minimal element with respect to $\sqsubseteq$.

ii. (5 points) Prove or disprove: Every nonempty subset of A (not necessarily finite!) has a minimal element with respect to $\sqsubseteq$.

P.T.O.

4. (10 points) Prove that if $n \in \mathbf{N}$, then the following program returns n^2.

```
SQUARE(n)
    x := 0
    k := 1
    loop
        exit when k = n + 1
        x := x + 2 * k - 1
        k := k + 1
    end loop
    return x
end SQUARE
```