1. a) $1^* \mid 1^*110(1^*110)^*111^*$

   b)

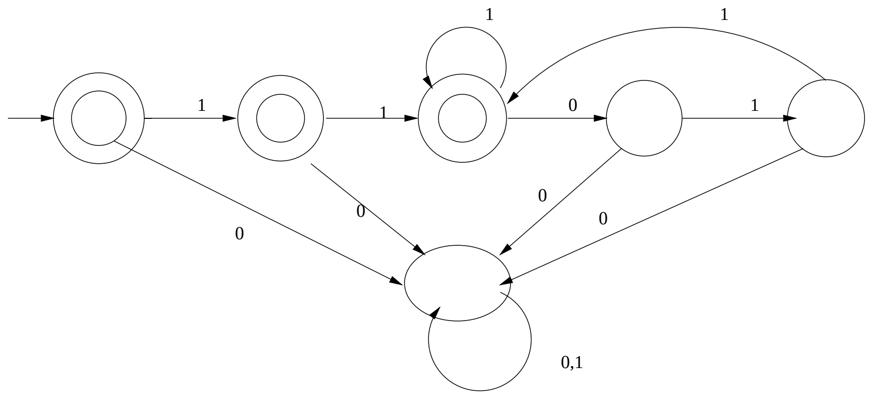

Figure 1: A det FSA corresponding to part a

2. a) Below is a description of the "most immediate" non deterministic FSA $\mathcal{M}$ corresponding to $(01^*)^*(0 \mid 1)$ (i.e. the one corresponding to the general transformation).

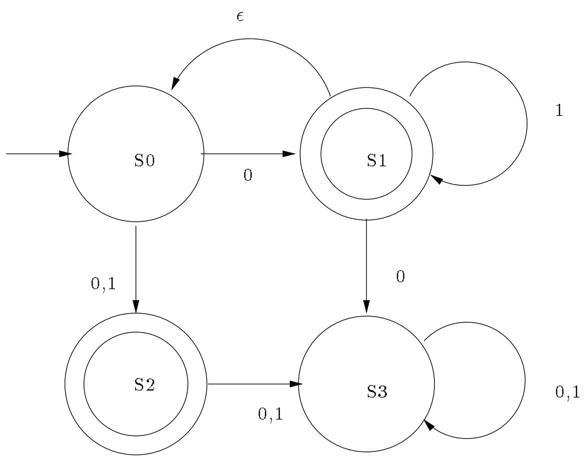

Figure 2: Immediate non det FSA

b) Using the non deterministic to deterministic transformation, we would obtain:

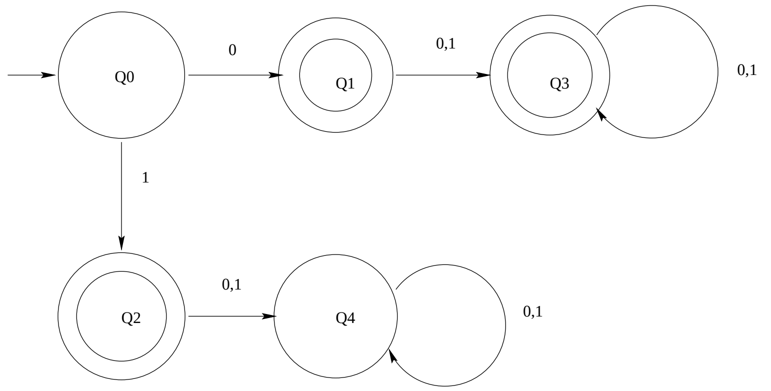

Figure 3: The resulting det FSA

which simplifies to

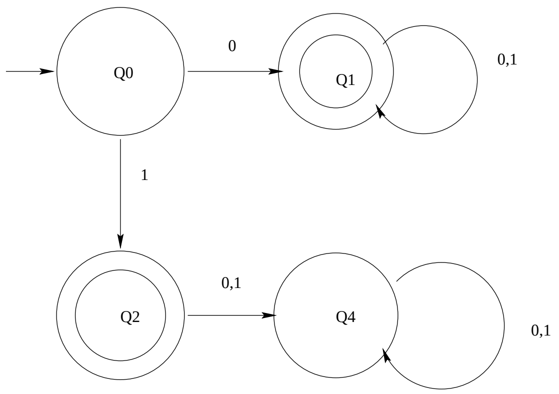

Figure 4: A reduced FSA

which corresponds to the equivalent regular expression: $(1 \mid 0(0 \mid 1)^*)$.

3. State q_1 is reachable precisely by strings in 11^*. State q_3 is reachable precisely by strings in $11^*(01)(01)^*$. It is easy to see that $11^* \mid 11^*(01)(01)^*$ is equivalent to $11^*(01)^*$.

4. There are at least two ways to prove this result.

   i) Following the printed hint, given (say, deterministic) $\mathcal{M} =< S, \delta, s_0, F >$ we construct non deterministic $\mathcal{M}^{REV} =< S \cup \{q_0\}, \delta^{-1}, q_0, \{s_0\} >$ where

   $$\delta^{-1}(q_0, \epsilon) = F$$
   $$\delta^{-1}(s, a) = \{s' \mid \delta(s', a) = s\}$$

   for $s \in S$, $a \in \Sigma$.

   For simplicity, we have not defined $\delta^{-1}(q_0, a)$ for $a \in \Sigma$. It can be defined as $\delta^{-1}(q_0, a) = \delta^{-1}(F) = \{s' \mid \delta(s', a) \in F\}$.

   ii) Following the hint in class, the result can be proved by induction on the depth or length of the construction of a regular expression R.

That is, we prove that if R is a regular expression, then there exists $R' : L(R') = (L(R))^{REVERSE}$.

(a) The basis is obvious with $R' = R$.

(b) If R is $R_1 \mid R_2$ then R' is $R_1^{'} \mid R_2^{'}$.
If R is $R_1 \cdot R_2$ then R' is $R_2^{'} \cdot R_1^{'}$
If R is $(R_1)^*$ then R' is $(R_1^{'})^*$.