**CSC 238S** **Term Test #2 Solutions** **Spring 1997**

1. a) Valid. Let $I$ be any interpretation in which $\forall x[P \vee Q]$ is true. If in this interpretation $\exists x[P]$ is true, then clearly $\exists x[P] \vee \forall x[Q]$ is true. Otherwise, $P$ is false for all domain elements so that in this interpretation $P \vee Q$ has the same truth value as $Q$ and therefore in order for $\forall x[P \vee Q]$ to be true, it must be the case that $\forall x[Q]$ is true.

   b) Not valid. Consider an interpretation $I$ in which there is exactly one domain element '$a$' such that $P(x)\mid_{x=a}$ is true and $Q$ is false for all domain elements.

   c) Not valid. Consider an interpretation with (only) two domain elements '$a$' and '$b$' such that $P(a)$ true, $Q(a)$ true, $R(a)$ false, $P(b)$ true, $Q(b)$ false, $R(b)$ true.

   d) Valid. Consider any interpretaiton and *any* domain element '$a$'. We have $P(a) \rightarrow Q(a) \wedge Q(a) \rightarrow R(a)$ and then (By propositional reasoning) $P(a) \rightarrow R(a)$.

2. a) $TH_3$ can be expressed as

$$x_1x_2x_3 \vee x_1x_2x_4 \vee x_1x_3x_4 \vee x_2x_3x_4$$

   b) $EQ_3$ cannot be expressed using only $\wedge$, $\vee$ since (as we proved) formulas using only $\wedge$, $\vee$ can only represent monotone Boolean functions and $EQ_3$ is not monotone since

   $EQ_3$ (TRUE, TRUE, TRUE, FALSE) = TRUE while

   $EQ_3$ (TRUE, TRUE, TRUE, TRUE) = FALSE.

3. Note: We are assumming that brother means full (rather than half) brother and similarly for uncle. a) $M(u) \quad \wedge M(v) \wedge \exists x_1 \exists x_2[PARENT(x_1, u) \wedge$
   $PARENT(x_2, u) \wedge PARENT(x_1, u) \wedge PARENT(x_2, v)]$

   b) $M(u) \quad \wedge \exists x[x \neq u \wedge PARENT(x, v) \wedge \exists w_1 \exists w_2$
   $PARENT(w_1, u) \wedge PARENT(w_1, x) \wedge$
   $PARENT(w_2, u) \wedge PARENT(w_2, x)]$

4. a) $\exists y[\neg(y = 0) \wedge ADD(u, y, v)]$

   b) $\exists z_1 \exists z_2[MULT(z_1, z_2, w) \wedge (1 < z_1) \wedge (z_1 < w)]$.