CSC 238S **Term Test #2** **March 17/97**

Name ______________________ Student No. ______________________

Tutorial: Name of Teaching Assistant and/or room number ______________________

Open Book
Answer ALL questions on test paper. Use backs of sheets for scratch work.
Total Marks: 50

[20] 1. For each of the following predicate formulas, determine whether or not the formula is valid. If valid, justify by explaining (in one or two English sentences) why the formula is true for an arbitary interpretation. (You may use known equivalences and known valid formulas.) If not valid, specify an interpretation for which the formula is false.

a) $\forall x[P \vee Q] \rightarrow (\exists x[P] \vee \forall x[Q])$

b) $(\exists x[P] \vee \forall x[Q]) \rightarrow (\forall x[P \vee Q] \vee \forall x[P \rightarrow Q])$.

c) $(\exists x[P \rightarrow Q] \wedge \exists x[Q \rightarrow R]) \rightarrow \exists x[P \rightarrow R]$.

d) $(\forall x[P \rightarrow Q] \wedge \forall x[Q \rightarrow R]) \rightarrow \forall x[P \rightarrow R]$.

[10] 2. Consider the following Boolean functions and determine whether or not the function can be expressed by a propositional formula using only the connectives $\wedge$, $\vee$. (Note that $\neg$ is not allowed.) Justify your answers.

a) $TH_3(x_1, x_2, x_3, x_4) = \begin{cases} \text{TRUE} & \text{if at least three of the } x_i \text{ are TRUE} \\ \text{FALSE} & \text{otherwise} \end{cases}$

b) $EQ_3(x_1, x_2, x_3, x_4) = \begin{cases} \text{TRUE} & \text{if exactly three of the } x_i \text{ are TRUE} \\ \text{FALSE} & \text{otherwise} \end{cases}$

[10] 3. Recall the predicates PARENT(x, y), $M(x)$, and $F(x)$ and their interpretation as in problem set 3. Again, assume the "biological laws" F_1, F_2, F_3 from problem set 3 which state that everyone is either male or female and that everyone has exactly two parents, one male and one female.

a) Construct a predicate formula $A(u, v)$ which expresses the relation that u and v are brothers.

b) Construct a predicate formula $A(u, v)$ which expresses the relation that u is the uncle of v.

[10] 4. Consider a predicate language with equality (=), the predicate symbols $\mathrm{ADD}(x, y, z)$ and $\mathrm{MULT}(x, y, z)$ and constant symbols a_0, a_1.

Consider the interpretation $\mathcal{I}$ with

$D = \mathbb{N} = \{0, 1, 2, \cdots\}$

$\mathcal{I}(\mathrm{ADD}(x, y, z))$ is $\{(a, b, c) | c = a + b\}$

$\mathcal{I}(\mathrm{MULT}(x, y, z))$ is $\{(a, b, c) | c = a \cdot b\}$

$\mathcal{I}(a_0)$ is 0, $\mathcal{I}(a_1)$ is 1.

Note that '=' is always interpreted as equality. Using this predicate language:

a) Construct a formula A with free variables u, v which expresses the binary relation $u < v$; that is, for the above interpretation $\mathcal{I}(A) = \{(a, b) | a < b\}$;

b) Construct a formula B with free variable w which expresses the compositeness relation; that is, $\mathcal{I}(B) = \{c | c$ is a composite ($=$ non prime) number$\}$.