CSC 238S **Term Test #1 Solutions** **Spring 1997**

1. a) testgrowth(1) = 1
   testgrowth(n) = 2 * testgrowth($n-1$) $- n + 2$ for $n > 1$

   b) It is easy to verify that testgrowth(i) $= i$ for $i = 1, 2, 3, 4, 5$.

   Claim: testgrowth(n) $= n$ for $n \geq 1$.

   c) Base case: testgrowth(1) = 1

   Induction step: Let $n > 1$

   Assume testgrowth($n-1$) $= n-1$
   Show testgrowth(n) $= n$

$$\begin{aligned}\text{testgrowth}(n) &= 2 * \text{testgrowth}(n-1) - n + 2 \\ &= 2 * (n-1) - n + 2 \\ &= n\end{aligned}$$

2. a) $T_1(0) = T_1(1) = 1$
   $T_1(n) = 2 + T_1(n-1)$

   b) $T_2(0) = T_2(1) = 1$
   $T_2(n) = 2 + T_2(n-1) + T_2(n-2)$

   c) It is easy to show that $T_1(n)$ is $\theta(n)$. (More precisely, $T_1(n) = 2n - 1$ for $n \geq 1$.) $T_2(n) \geq fib(n)$ for $n \geq 1$ and $fib(n)$ is $\Theta(\alpha^n)$ where $\alpha = \frac{1+\sqrt{5}}{2}$. Therefore, $T_2(n)$ is exponential in n while $T_1(n)$ is linear so that T_2 will be much larger than $T_1(n)$ where n is large.

3. $a)i)$ $((A \vee B) \rightarrow (C \wedge D))$
   $ii)$ $C \rightarrow E$
   $iii)$ $\neg E \rightarrow A$

   b) iv) is expressed as E. We want to argue that E is a logical consequence of the three statements in part a).

   Here is a "semi-formal" proof

$\neg E \rightarrow A$	assumption
$C \rightarrow E$	assumption
$\neg E \rightarrow \neg C$	(equivalent to $C \rightarrow E$)
$(A \vee B) \rightarrow (C \wedge D)$	assumption
$\neg(C \wedge D) \rightarrow \neg(A \vee B)$	equivalent to above implication
$(\neg C \vee \neg D) \rightarrow (\neg A \wedge \neg B)$	using De Morgan's Laws
$\neg C \rightarrow (\neg A \wedge \neg B)$	if $F \vee G \rightarrow H$ then $F \rightarrow H$
$\neg C \rightarrow \neg A$	if $F \rightarrow G \wedge H$ then $F \rightarrow G$
$\neg E \rightarrow \neg A$	since $\neg E \rightarrow \neg C$ and $\neg C \rightarrow \neg A$
$\neg E \rightarrow A \wedge \neg A$	since $\neg E \rightarrow A$ and $\neg E \rightarrow \neg A$
$\neg E \rightarrow FALSE$	$A \wedge \neg A$ is always false.