CSC 238S **Term Test #1** **February 10/97**

Name ______________________________ Student No. ______________________________

Tutorial: Name of Teaching Assistant and/or room number ______________________

Open Book
Answer ALL questions on test paper. Use backs of sheets for scratch work.
Total Marks: 30 (not counting bonus)

Consider the following functions:

```
function testgrowth(n:int)
   if n <= 1 then result 1
   else result
      2 - n + 2 * testgrowth(n-1)
   endif
end testgrowth
```

```
function fib(n: int)
   if n <= 1 then result 1
   else result
       fib(n-1) + fib(n-2)
   endif
end fib
```

[4] 1. a) Write a recurrence that characterizes the value of testgrowth(n) for $n \geq 1$.

[4] b) Compute the value of testgrowth(n) for several small values of n; e.g. $n = 1, 2, 3, 4, 5$. State an explicit (i.e. non recursive) formula $t(n)$ for testgrowth(n) for $n \geq 1$.

[4] c) Prove by induction that testgrowth(n)=$t(n)$ for $n \geq 1$.

[4] 2. a) Write a recurrence for $T_1(n)$ for $n \geq 0$, where $T_1(n)$ is the number of instructions executed by testgrowth(n). To be precise, count each 'if' statement as one instruction and each 'else' statement as one instruction (not counting the recursive calls).

[4] b) Write a recurrence for $T_2(n)$ for $n \geq 0$, where $T_2(n)$ is the number of instructions executed by fib(n).

[4] c) For large n, argue briefly (but convincingly) that testgrowth(n) will execute less instructions than fib(n).

3. Consider the following dialogue:

ACTOR 1 makes the following statements:

Si) If either I or my best friend will understand function testgrowth, then we will both understand function fib.

Sii) If I will understand function fib, then I will understand everthing in this course.

Siii) If I will not understand everything in this course, then at least I will understand function testgrowth.

ACTOR 2 replies:

Siv) So you will understand everything in this course.

Let A be the proposition "I will understand function testgrowth".

B is the proposition "my best friend will understand function testgrowth".

C is the proposition "I will understand function fib".

D is the proposition "my best friend will understand function fib".

E is the proposition "I will understand everything in this course".

[6] a) Express statements Si, Sii, Siii as propositional formulas using the propositional variables A, B, C, D, E.

[2] b) Bonus: Explain using propositional reasoning why the reply of ACTOR 2 is logical.