CSC238S **Problem Set 3 Solutions** **Spring 1997**

1 (a) $\forall x[E(x) \rightarrow \neg C(x)]$

b) $\forall x[\forall y F(x,y) \rightarrow \neg E(x)]$

c) $F(a,b) \wedge \neg F(b,a)$

2 (a) i) $\forall x \forall x[C(x,y) \rightarrow S(x,y)]$

ii) $\forall x[\forall y[\neg S(x,y)] \rightarrow T(x)]$

iii) $\forall x \forall y[\exists z[T(z) \wedge A(x,y,z)]]) \rightarrow \neg C(x,y)]$

iv) $\exists x S(x, JR) \rightarrow (S(JJ, JR) \vee S(CJ, JR))$

v) $\forall y[S(CJ,y) \rightarrow \exists z A(CJ,y,z)]$

vi) $\forall z[A(CJ, JR, z) \rightarrow \forall y \neg S(z,y)]$

vii) $\exists x C(x, JR) \rightarrow C(JJ, JR)$

2 (b) vii) does not logically follow. As explained in class we would have to reinterpret iv) as

$$iv)\ \forall x[Sx, JR) \rightarrow (x = JJ \vee x = CJ)]$$

in order to have vii) logically follow i) - vi).

3 (a) i) Does not logically follow! Consider an interpretation in which a brother and sister produce a child (this is not socially or legally acceptable but it is logically and biologically possible).

ii) Does not logically follow! As in real life, Ed and Jack may be cousins by virtue of having one set of grandparents in common and Jack and Susie may have a different set of grandparents in common.

iii) Logically follows!
PARENT (a,a) implies ANCESTOR(a,a) by law iv). However, this is leads to a contradiction as by law vi) ANCESTOR (a,a) $\rightarrow$ $\neg$ ANCESTOR (a,a) which implies $\neg$ ANCESTOR (a,a).

3 (b) Consider any individual a_0 and for $i \geq 1$ let a_i be a parent of a_{i-1}. If the domain is finite, then for some $i < j$, $a_i = a_j$. But aj is (by induction) an ancestor of a_i and hence ANCESTOR (a_j, a_i) and thus ANCESTOR (a_i, a_i) which contradicts law vi).