CSC238S **Problem Set 3** **Due: March 17/97**

1. Consider an interpetation in which the domain is the set of all programs and the predicate $F(x, y)$ denotes "program x is at least as fast as program y", the predicate $C(x)$ denotes "program x is correct", and the predicate $E(x)$ denotes "program x is easy to understand". Let a and b be constant symbols referring to particular members of the domain of interpretation. Given **only** the predicates as above, express the following English sentences in prediate logic.

   (a) Any program that is easy to understand is not correct.

   (b) The fastest programs are correct, but are not easy to understand.

   (c) Program a is faster than program b.

2. **(a)** Following are seven statements regarding people and certain individuals. . You are to express each of them in predicate logic. The table below lists the constant and predicate symbols in the first-order language which you are to use, along with their intended meanings:

Constant Symbol	Intended Meaning	Predicate Symbol	Intended Meaning
JJ	"Jesse James"	S(x, y)	"x is suspected of the robbery of y"
CJ	"Calamity Jane"	C(x, y)	"x is convicted for the robbery of y"
JR	"John D. Rockefeller"	T(x)	"x is trustworthy"
		A(x, y, z)	"x has an alibi for the robbery of y supported by z"

Example: Anyone who is trustworthy has not been convicted of anyone's robbery.

Translation to predicate logic: $(\forall x)[T(x) \rightarrow \neg(\exists y)[C(x, y)]$.

(i) Nobody is convicted for somebody's robbery if he/she is not suspected of it.

(ii) Anyone who is not suspected of any robbery is trustworthy.

(iii) Anyone who is suspected of someone's robbery, but has an alibi for that robbery supported by a trustworthy person, is not convicted for that robbery.

(iv) If anyone is suspected of John D. Rockefeller's robbery, then Jesse James or Calamity Jane are suspected.

(v) For any robbery of which Calamity Jane is suspected, she has an alibi supported by someone.

(vi) Anyone who supports Calamity Jane's alibi for the robbery of John D. Rockefeller is not suspected of anyone's robbery.

(vii) If anyone is convicted for the robbery of John D. Rockefeller, then Jesse James is convicted.

(b) Referring to the statements in part (a), prove (by giving a careful argument in English) that the conjunction of (i)–(vi) logically implies (vii).

3. Assume we have a non empty set of individuals as a Domain, and that the following symbols exist and are interpreted as follows.

predicates:	PARENT(X,Y)	signifies	"X is a biological parent of Y"
	ANCESTOR(X,Y)	signifies	"X is a biological ancestor of Y"
	M(X)	signifies	"X is a male"
	F(X)	signifies	"X is a female"

Consider each of the following "biological laws":

i) F_1 $\quad \forall X[(\mathrm{M}\ (X) \vee \mathrm{F}\ (X)) \wedge (\mathrm{M}(X) \rightarrow \neg \mathrm{F}(X))]$
ii) F_2 $\quad \forall Y \exists X_1 \exists X_2[\mathrm{PARENT}\ (X_1,Y) \wedge \mathrm{PARENT}\ (X_2,Y) \wedge \mathrm{M}\ (X_1) \wedge \mathrm{F}\ (X_2)]$
iii) F_3 $\quad \forall X_1 \forall X_2 \forall Y[(\mathrm{PARENT}\ (X_1,Y) \wedge \mathrm{PARENT}\ (X_2,Y) \wedge X_1 \neq X_2)$
$\rightarrow ((\mathrm{M}\ (X_1) \rightarrow \mathrm{F}\ (X_2)) \wedge (\mathrm{F}\ (X_1) \rightarrow \mathrm{M}\ (X_2)))]$
iv) F_4 $\quad \forall X \forall Y[\mathrm{PARENT}(X,Y) \rightarrow \mathrm{ANCESTOR}(X,Y)]$
v) F_5 $\quad \forall X \forall Y \forall Z[(\mathrm{PARENT}(X,Y) \wedge \mathrm{ANCESTOR}(Y,Z)) \rightarrow \mathrm{ANCESTOR}(X,Z)]$
vi) F_6 $\quad \forall X \forall Y[\mathrm{ANCESTOR}(X,Y) \rightarrow \neg \mathrm{ANCESTOR}(Y,X)]$

Consider the following definitions:

GRANDPARENT (X,Y) :

$$\exists Z\,(\mathrm{PARENT}\ (Z,Y) \wedge \mathrm{PARENT}\ (X,Z))$$

COUSIN (X,Y):

$$\exists Z\,(\mathrm{GRANDPARENT}\ (Z,X) \wedge \mathrm{GRANDPARENT}\ (Z,Y))$$

(a) Which of the following logically follows from the above biological laws ; i.e. in any interpretation making all of the axioms true, the statement is true. Explain your answer briefly. Note that an interpretation need not correspond to "reality".

i) An individual has at least 3 (different) grandparents

ii) The COUSIN relation is transitive; i.e.

$$\forall X \forall Y \forall Z\,(\mathrm{COUSIN}\ (X,Y) \wedge \mathrm{COUSIN}\ (Y,Z)) \rightarrow \mathrm{COUSIN}\ (X,Z)$$

iii No one is their own parent; i.e.

$$\forall X \neg\ \mathrm{PARENT}\ (X,X)$$

(b) For any interpretation satisfying the above biological laws, show that the domain must be infinite. (Note: this exercise is just an exercise and is not meant to challenge any theory of creation.)