CSC238S **Problem Set 2** **Due: February 24/97**

1. Let $T, g : \mathbb{N} \to \mathbb{N}$ be functions that satisfy

   a) $T(b^k)$ is $\Omega(g(b^k))$ for all k

   b) T and g are nondecreasing

   c) $g(c \cdot n)$ is $\Omega(g(n))$, for any constant $c > 0$.

   Show $T(n)$ is $\Omega(g(n))$.

2. a) Show $\log(n!)$ is $\Omega(n \log n)$ and conclude from page 78 that $\log(n!)$ is $\Theta(n \log n)$.

   b) Show $n!$ is not $O(3^n)$.

3. Consider the following recurrence:

   $T(0) = 0$

   $T(n) = T(\lfloor n/2 \rfloor) + T(\lfloor n/4 \rfloor) + n$ for $n \geq 1$

   where $\lfloor x \rfloor$ is the floor of x.

   Prove by induction that $T(n)$ is $O(n)$.

   Hint: Use the method of "Solving Recurrences by Guessing a Solution" and note that $c(\lfloor x \rfloor) \leq cx$ for any positive constant c.

4. Problem 6 on page 15 of Gersting.

5. Let $\oplus$ represent the exclusive OR connective.

   (a) Construct a propositional formula using only $\oplus$ which is logically equivalent to FALSE.

   (b) Construct a propositional formula Q using only the connectives $\oplus$ and $\rightarrow$ and the propositonal variable x such that Q is logically equivalent to $\neg x$.

   (c) Using known equivalences and the fact that $\{\wedge, \vee, \neg\}$ is functionally complete, show $\{\oplus, \rightarrow\}$ is functionally complete.

6. Let Q be a propositional formula using the connectives $\vee$ and $\rightarrow$, one propositional variable x and the constant symbol TRUE. Show by induction on the length of formulas that Q is either logically equivalent to x or is logically equivalent to TRUE. (We could use this fact to conclude that $\{\vee, \rightarrow\}$ is not functionally complete.)