CSC238S **Problem Set 1** **Due: January 27/97**

1. a) Find a formula for
$$\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\cdots+\frac{1}{n(n+1)}$$

   b) Prove the correctness of this formula by induction.

2. Consider the following (selection sort) procedure:

```
procedure sort (var A : array 1 .. * of int)
    const n : int := upper(A)      % Note that n >= 1
    var i : int := 1

    loop
        % Loop invariants would go here
        exit when i = n
        % 1 <= i < n
        var candidate : int := i
        for j : i + 1 .. n
            if A (j) < A (candidate) then
                candidate := j
            end if
        end for
        % max-inv : A [candidate] <= A[j] for all j : i <= j <= n
        const temp : int := A (candidate)
        A (candidate) := A (i)
        A (i) := temp

        i := i + 1
    end loop

end sort
```

   Prove (by induction) that the following statements are loop invariants for this program.

   S1: if $i > 1$ then $A[1] \leq A[2] \leq \cdots \leq A[i-1]$

   S2: if $i > 1$ then $A[i-1] \leq A[k]$ for all k such that $i \leq k \leq n$

   S3: $A[1..n]$ contains exactly the same elements as $A_0[1..n]$, possibly rearranged.

   Hint: Don't try to prove each statement individually. You may use the fact that "max-inv" is an invariant.

3. Consider the recurrence $a_n = 2a_{n-1} + 3a_{n-2}$ (for $n \geq 2$) with $a_0 = 1$, $a_1 = 7$.

   a) Derive an explicit (i.e. non recursive) formula for a_n, $n \geq 0$.

   b) Prove by induction that the formula is correct.