Due: Wednesday, October 15, 3PM EST

This assignment is worth 15% of the final grade. Each question or subquestion is worth a multiple of 5 points. If you have no idea how to answer a question (or part of a question), you will receive 20% of the credit for that question (or subquestion) by leaving the question (or subquestion) blank. If your answer makes no sense, you will not receive any credit. Any answer that shows some understadning of the question will receive some credit.

1. (10 points) Consider the following question related to hashing and has tables. Let $h(ID) = ID(mod\ 83)$. By "mod 83", I mean divide by 83 and take the remainder. For example, for the ID = 1021518, we would get $1021518(mod\ 83) = 37$. That is, this ID is being hashed or mapped onto a much smaller number (i.e. a number between 0 and 82).

   Suppose the above hash function was a perfect hash function (which it is not) in the sense that it is equivalent to a balls and bins experiement where every ball lands in a bin chosen uniformly at random. Suppose there were only 3 students in our class and all student IDs were integers x with $1 \leq x \leq 1200$.
   Calculate the probability that (at least) two students will be hashed to the same hash value. Hint: First calculate the probability that all 3 students get a unique hash value.

2. A simple graph $G = (V, E)$ has a k-colouring is there exists a function $\chi : V \rightarrow \{1, 2, \ldots, k\}$ such that $\chi(u) \neq \chi(v)$ for all $(u, v) \in E$. The graph colouring decision problem is "Given a graph $G = (V, E)$ and an integer k, decide if G has a k-colouring. The graph colouring problem is an NP-complete problem for $k \geq 3$.
   Note: You do not need to know anything about NP-complete problems other than that it is widely believed that such a problem cannot be done efficiently.
   However, for certain classes of graphs, the graph colouring question can be answered efficiently.

   - (5 points) For a fixed k, roughly estimate the time it would take to decide if $G = (V, E)$ has a k-colouring if you wanted to naively try all possible functions $\chi : V \rightarrow \{1, 2, \ldots, k\}$. State your time estimate in terms of $n = |V|, m = |E|$, and k.
   - (5 points) A bipartite graph $G = (V, E)$ where $V = V_1 \cup V_2$, $V_1 \cap V_2 = \emptyset$, and $E = (V_1 \times V_2)$. That is the vertices are partitioned into two disjoint sets V_1 and V_2, and all edges are between vertices in V_1 and V_2.
     Show that every bipartite graph is 2-colourable.
   - (5 points) Suppose $G = (V, E)$ is a tree. That is, G is connected and has no cycles. Show that every tree is a bipartite graph

3. (5 points) Suppose $L_1 \leq_\tau L_2$ via a computabale transformation f and $L_2 \leq_\tau L_3$ via a computabale transformation g. In class we claimed by transitivity that we

then have $L_1 \leq_\tau L_3$. Indicate a function h such that $L_1 \leq_\tau L3$ via the computabale transformation h.

4. (10 points)

   Let $t(n) = 2^n + n$. Consider the language $L = \{(M, w)|\text{M halts on } w \text{ in at most } t(n) \text{ steps where } n = |w|\}$. More formally, $L = \{<< M >, w > |\text{M halts on } w \text{ in at most } t(n) \text{ steps where } n = |w|\}$. Is the language L a decideable language? Explain in a few sentences.