Zero-Knowledge Proofs

Akshayaram Srinivasan

Gödel prize 1993: László Babai, Shafi Goldwasser, Silvio Micali, Shlomo Moran, and Charles Rackoff

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Turing Award 2012: Shafi Goldwasser and Silvio Micali

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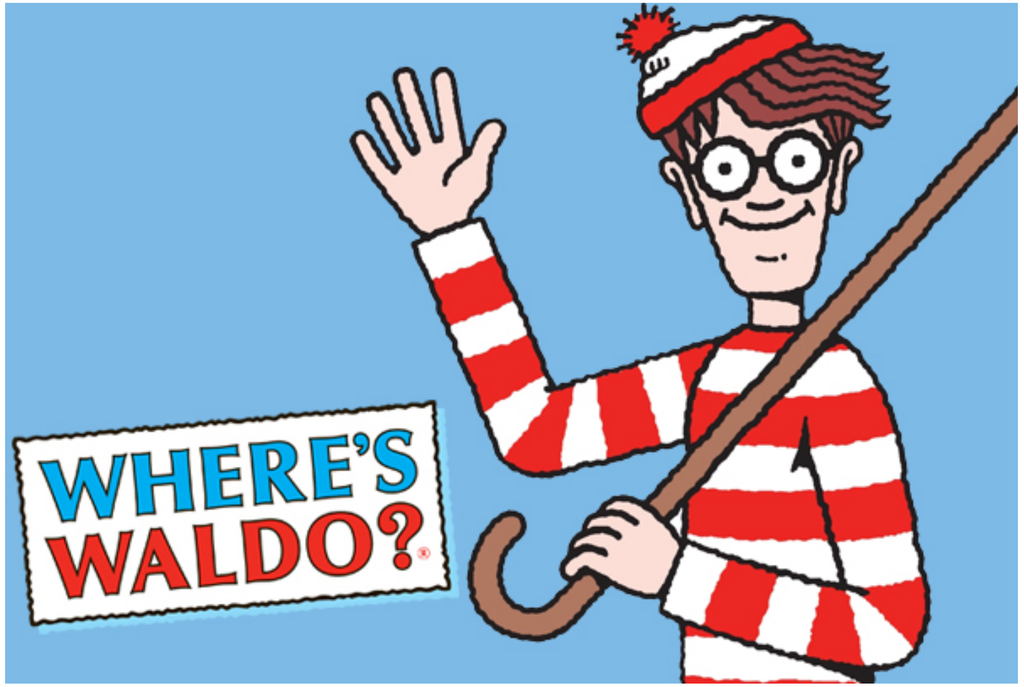
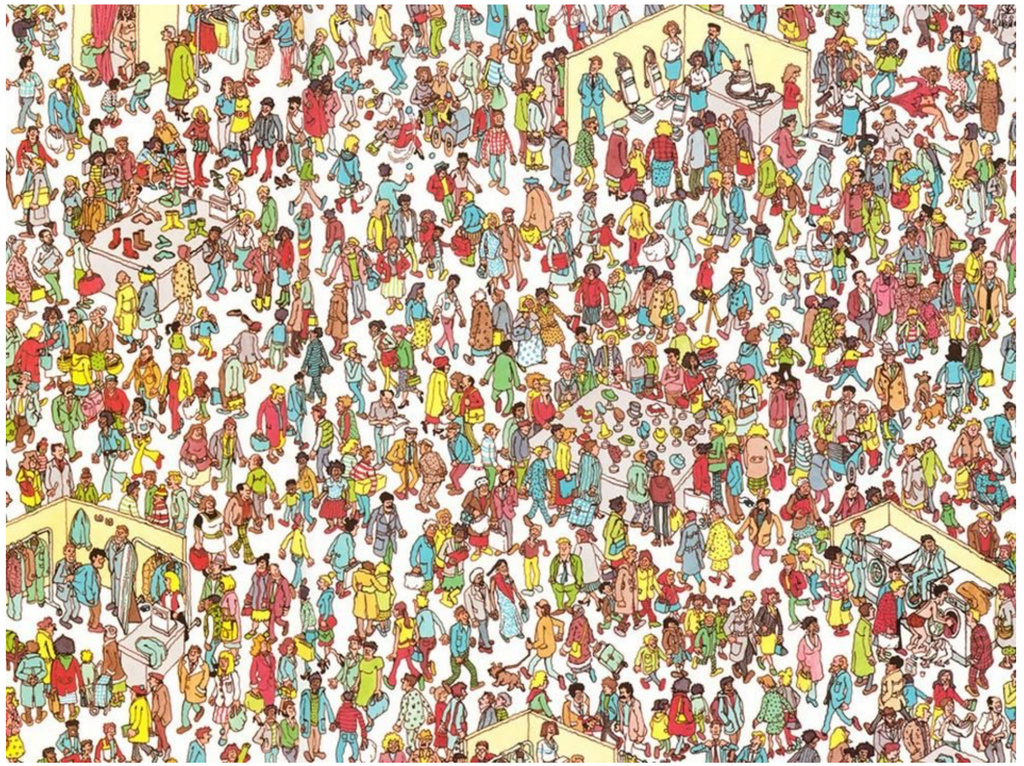
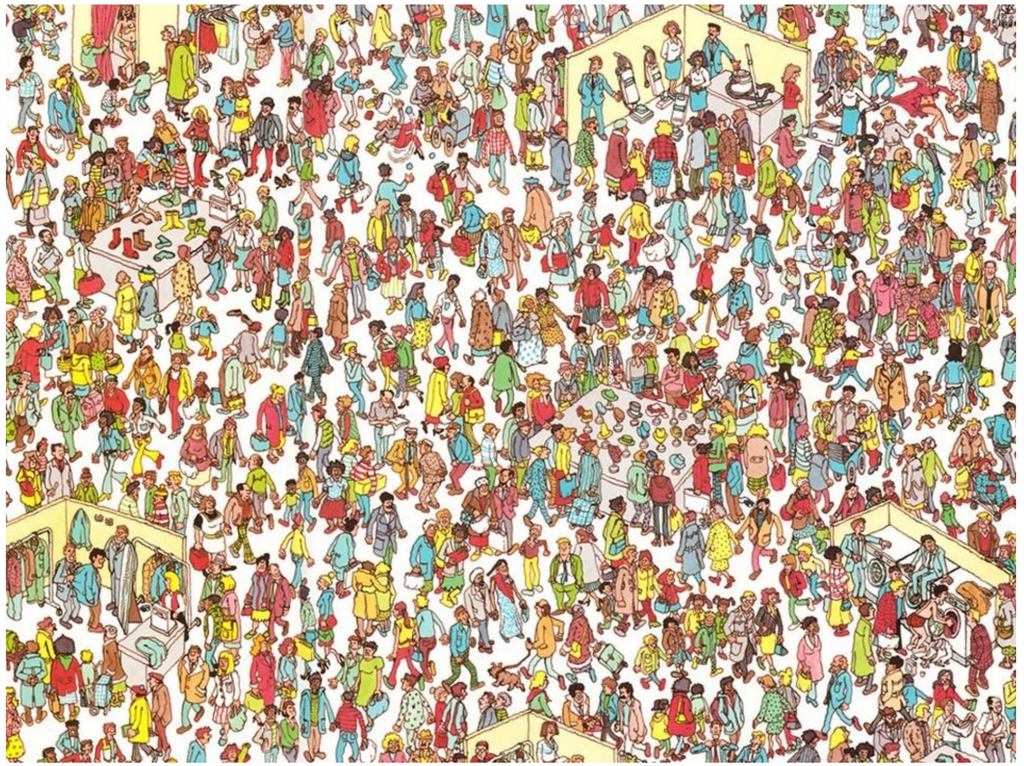
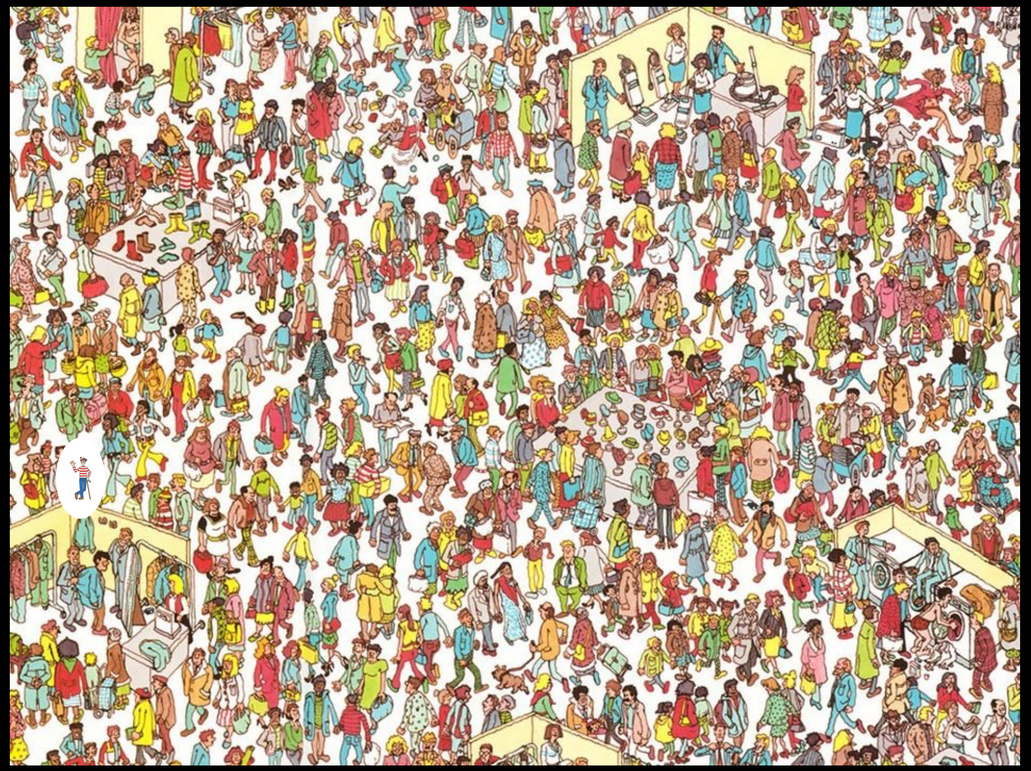
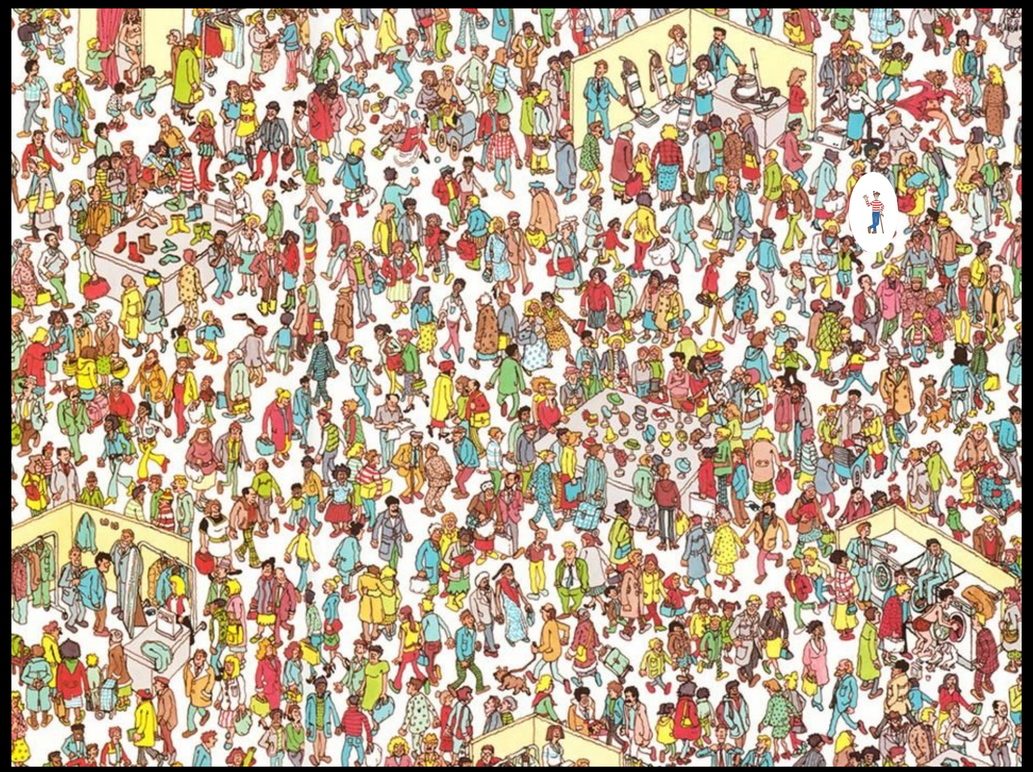

WHERE'S
WALDO?

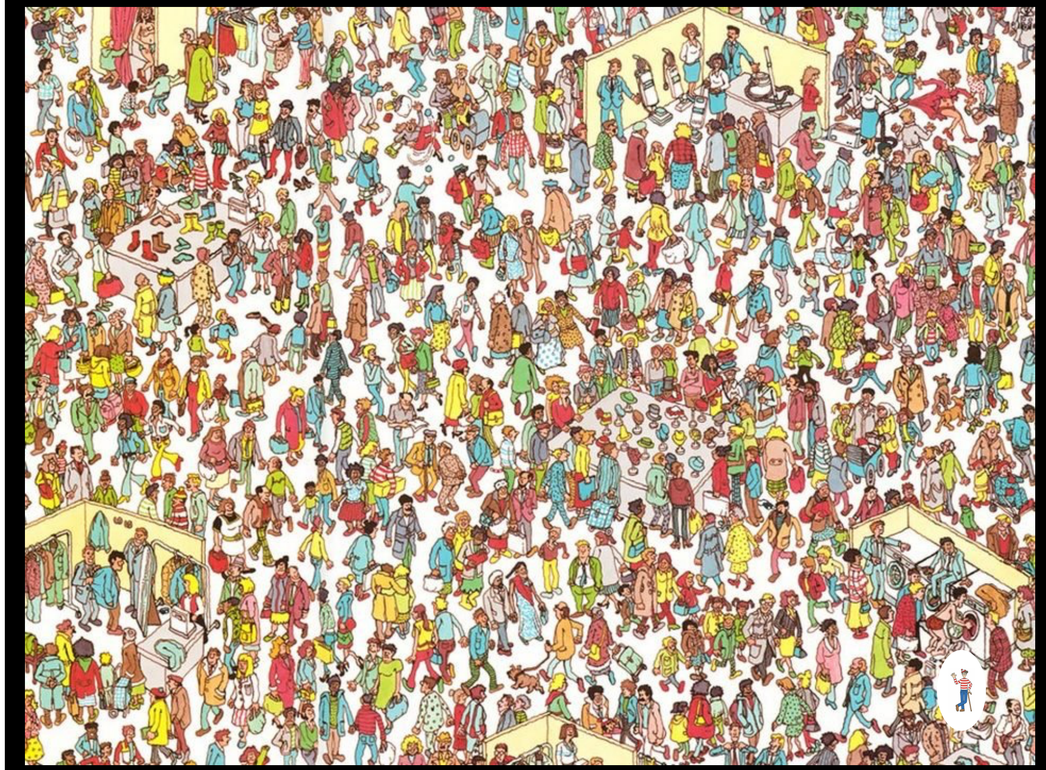




Can I prove where's Waldo without revealing his coordinates?







Example

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You were convinced about an Assertion

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You didn’t learn any other information apart from the fact that the assertion is true.

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This is the core idea behind zero-knowledge proofs

Interactive Proofs

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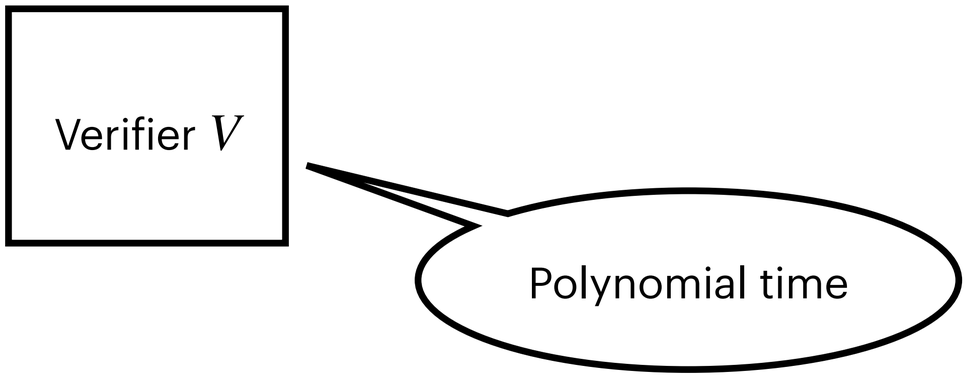
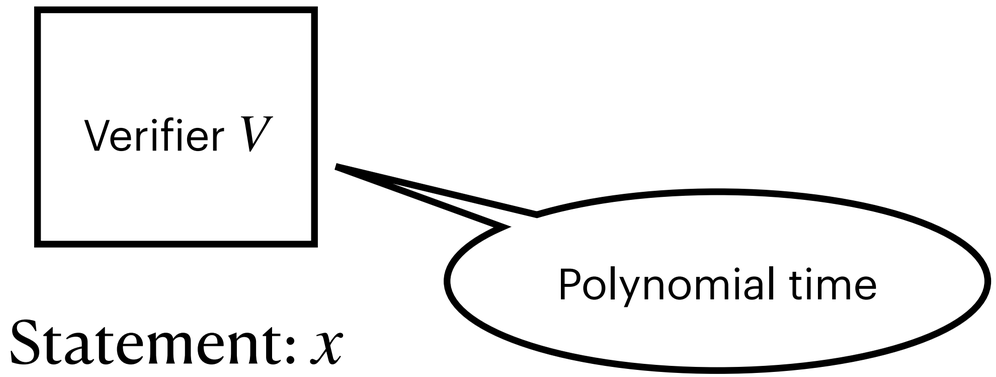


Interactive Proofs



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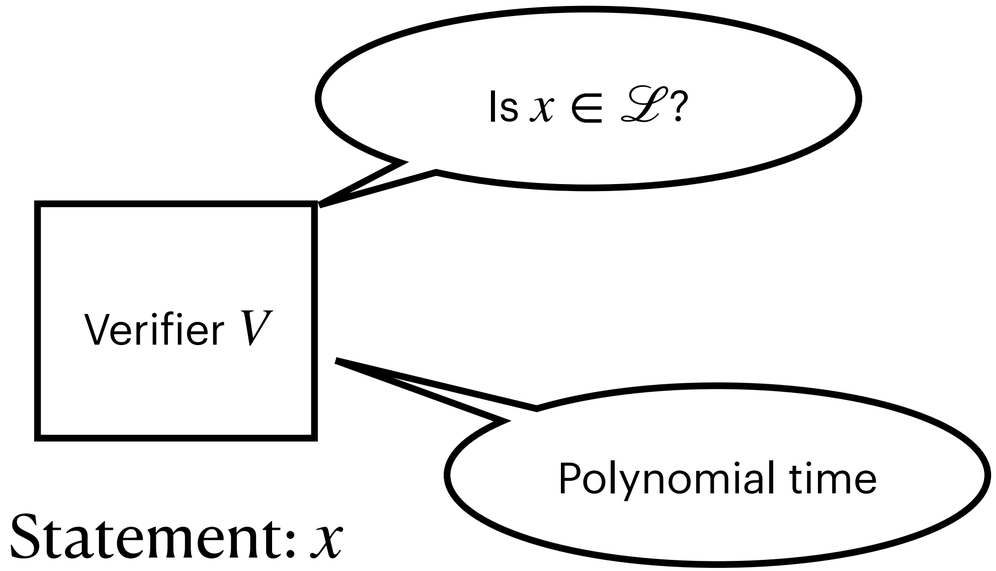

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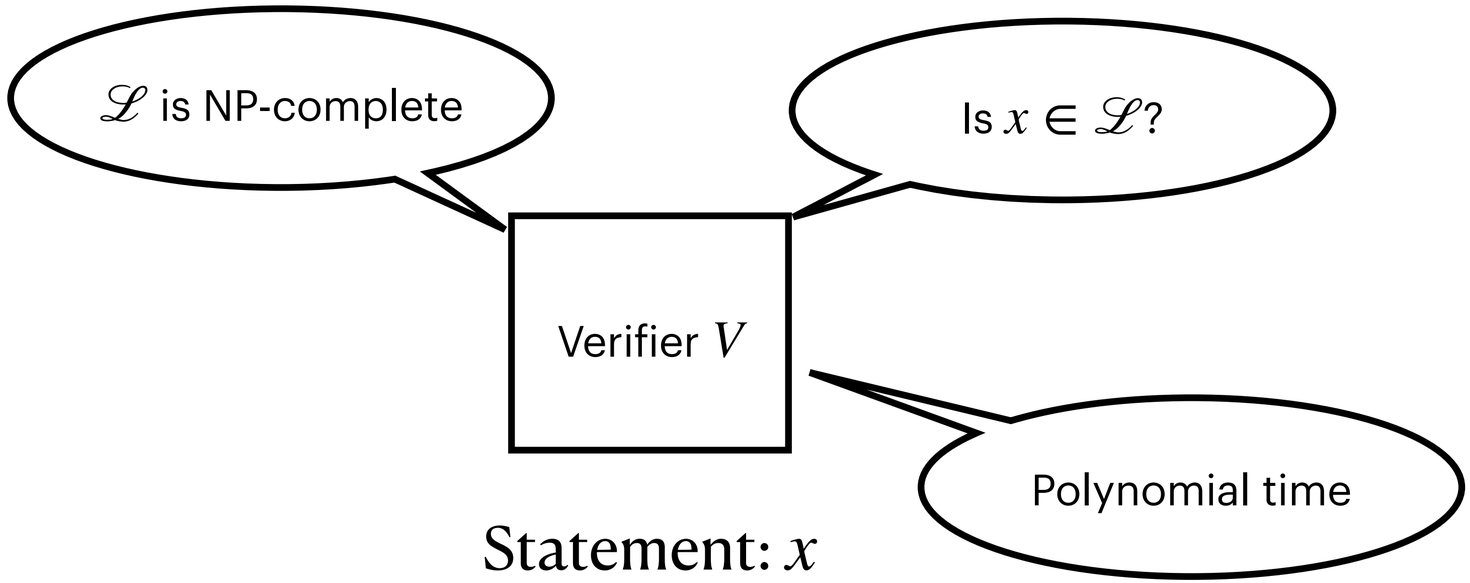

Interactive Proofs

Statement: x

Interactive Proofs

Interactive Proofs

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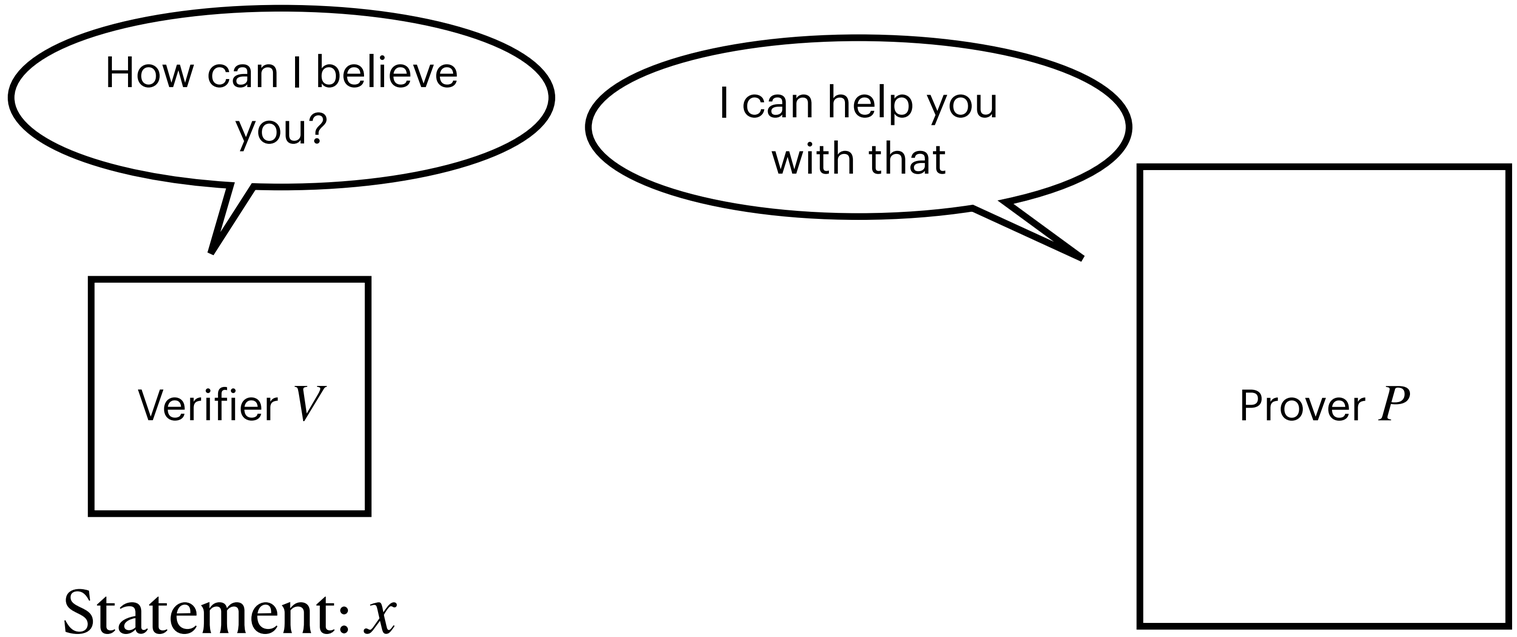

Interactive Proofs

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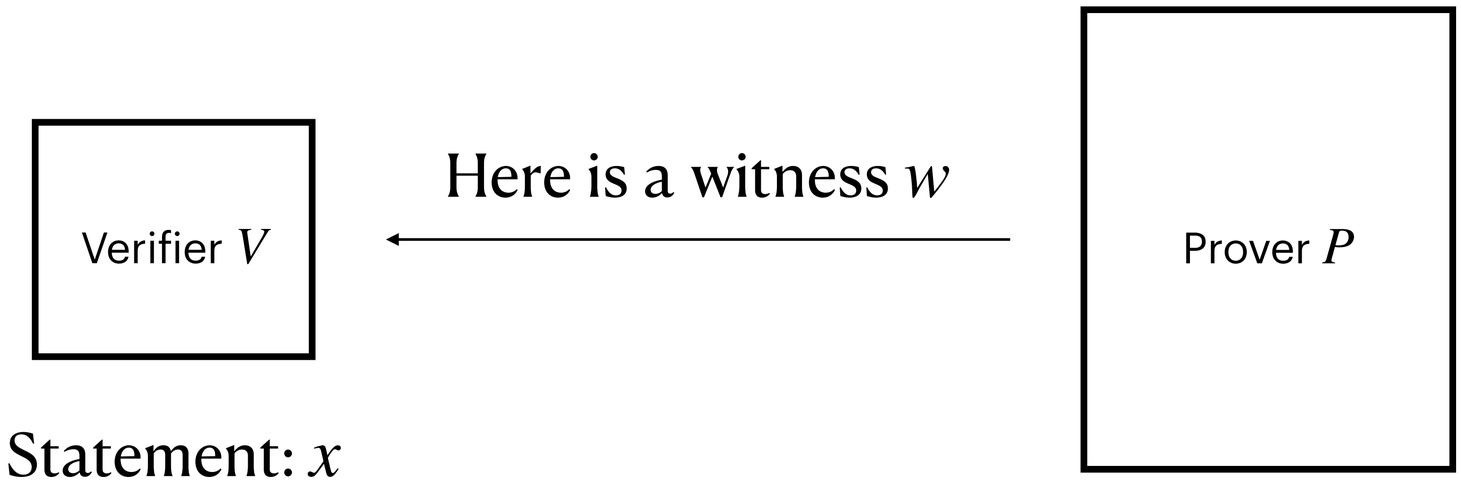

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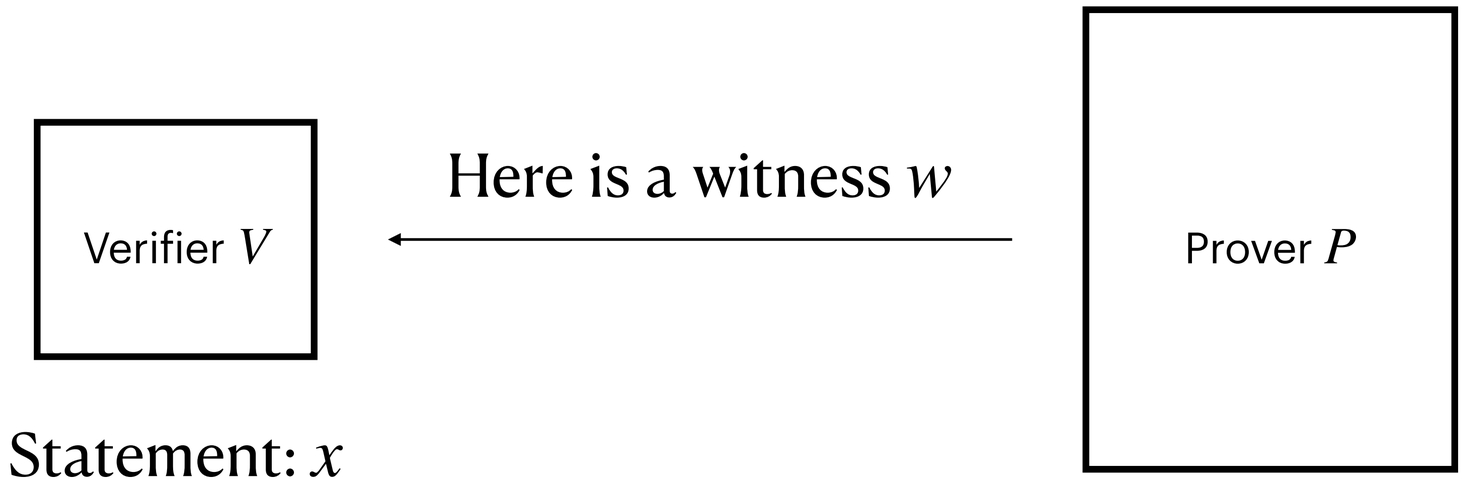
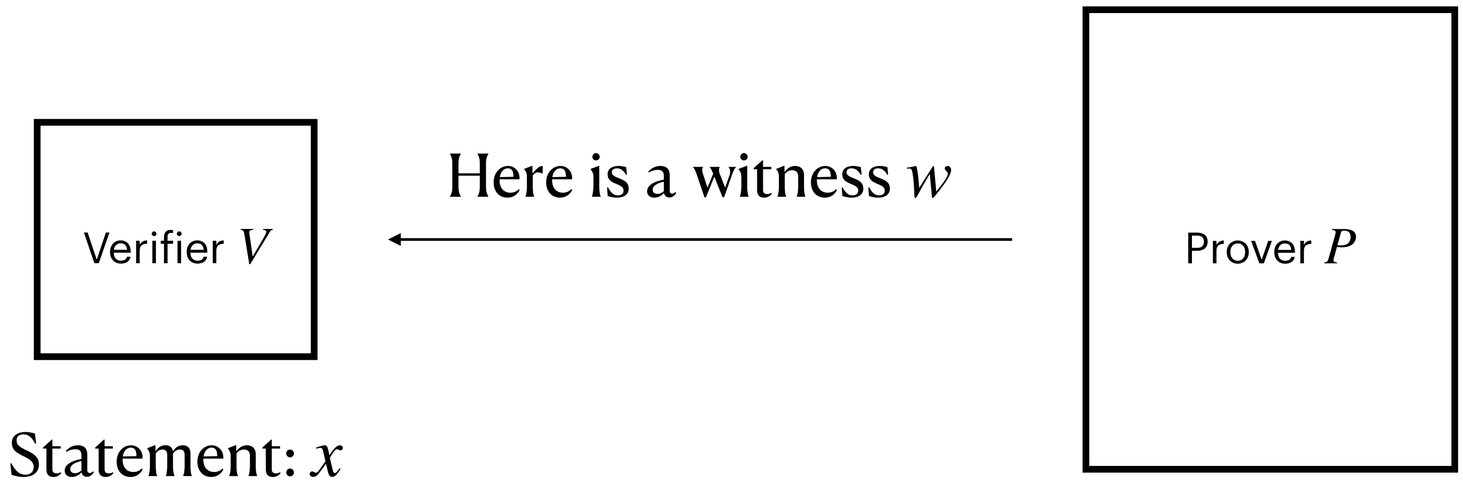


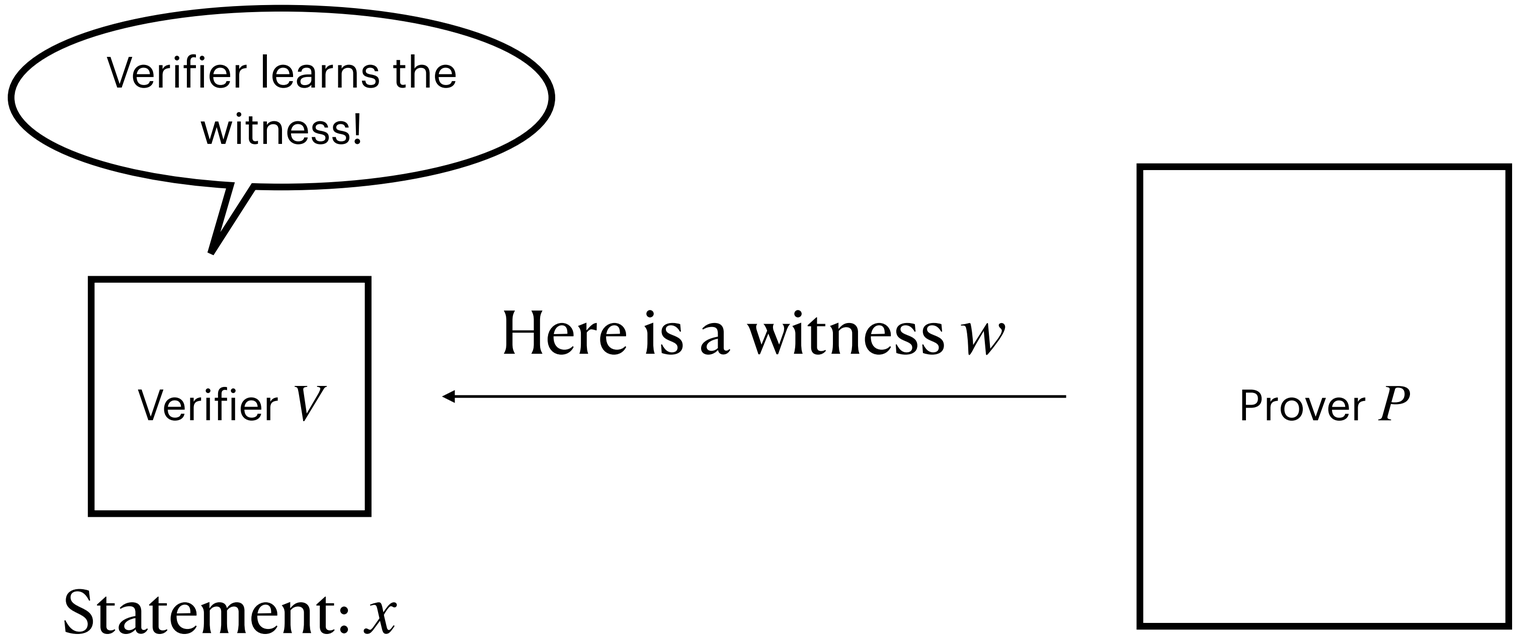
- **Completeness:** If $x \in \mathscr{L}$, then prover can send a witness that makes the verifier accept.

Interactive Proofs



- **Completeness:** If $x \in \mathscr{L}$, then prover can send a witness that makes the verifier accept.
- **Soundness:** If $x \notin \mathscr{L}$, then whatever string prover sends, the verifier rejects.

Interactive Proofs



- **Completeness:** If $x \in \mathscr{L}$, then prover can send a witness that makes the verifier accept.
- **Soundness:** If $x \notin \mathscr{L}$, then whatever string prover sends, the verifier rejects.

Can we convince the verifier that $x \in \mathscr{L}$ without leaking any information about the witness?

Zero-Knowledge Proofs

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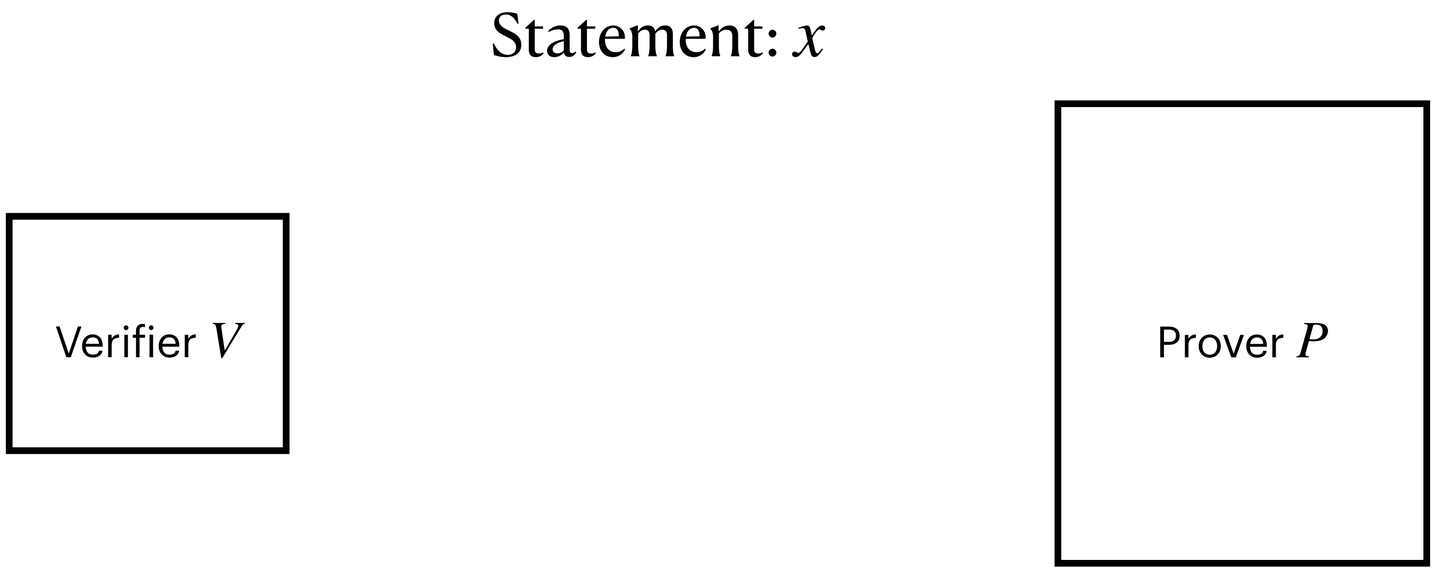

Zero-Knowledge Proofs

Statement: x

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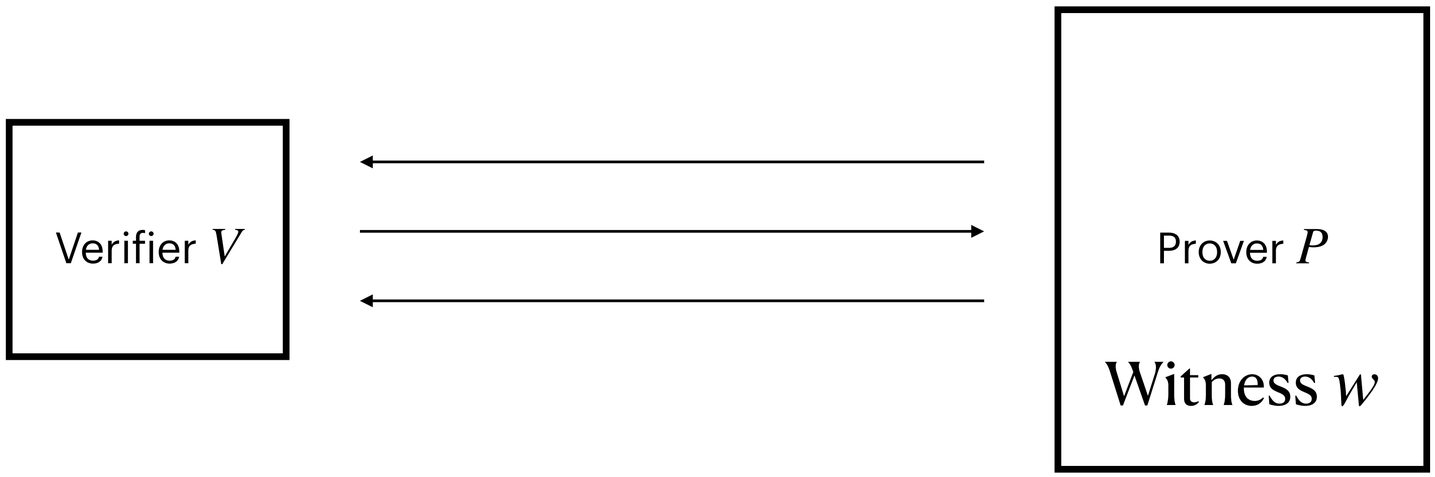

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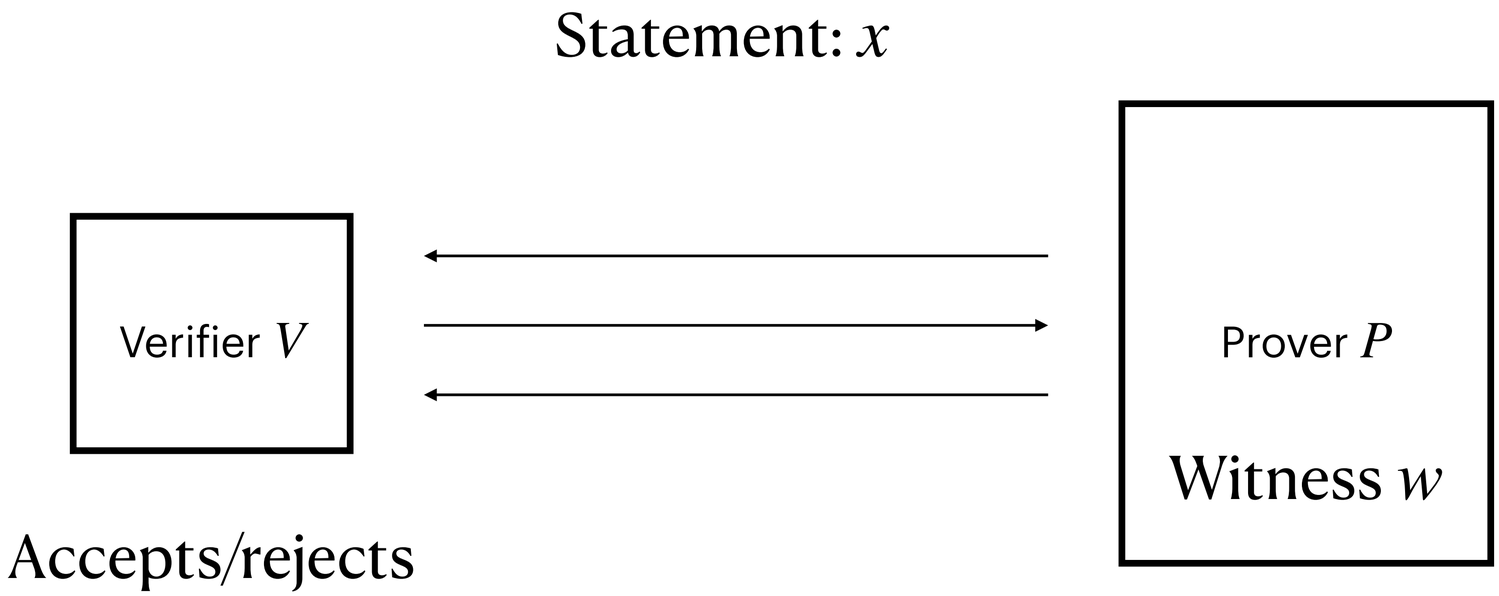

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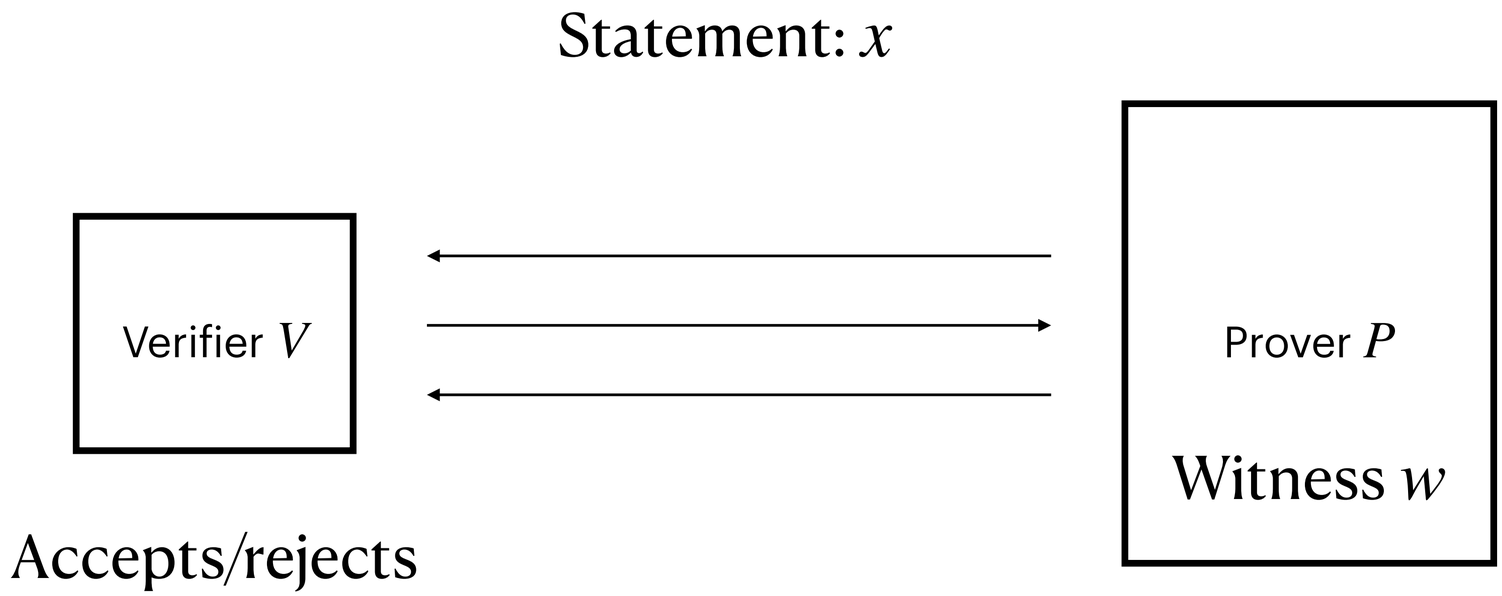
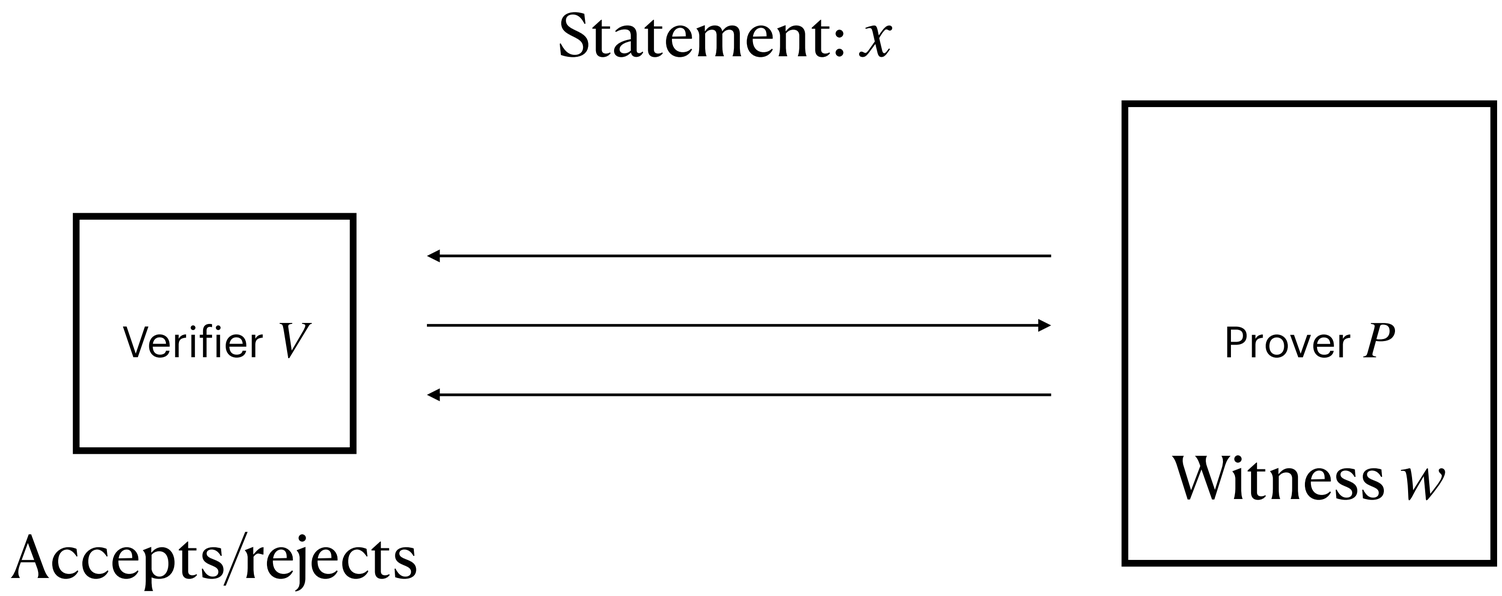


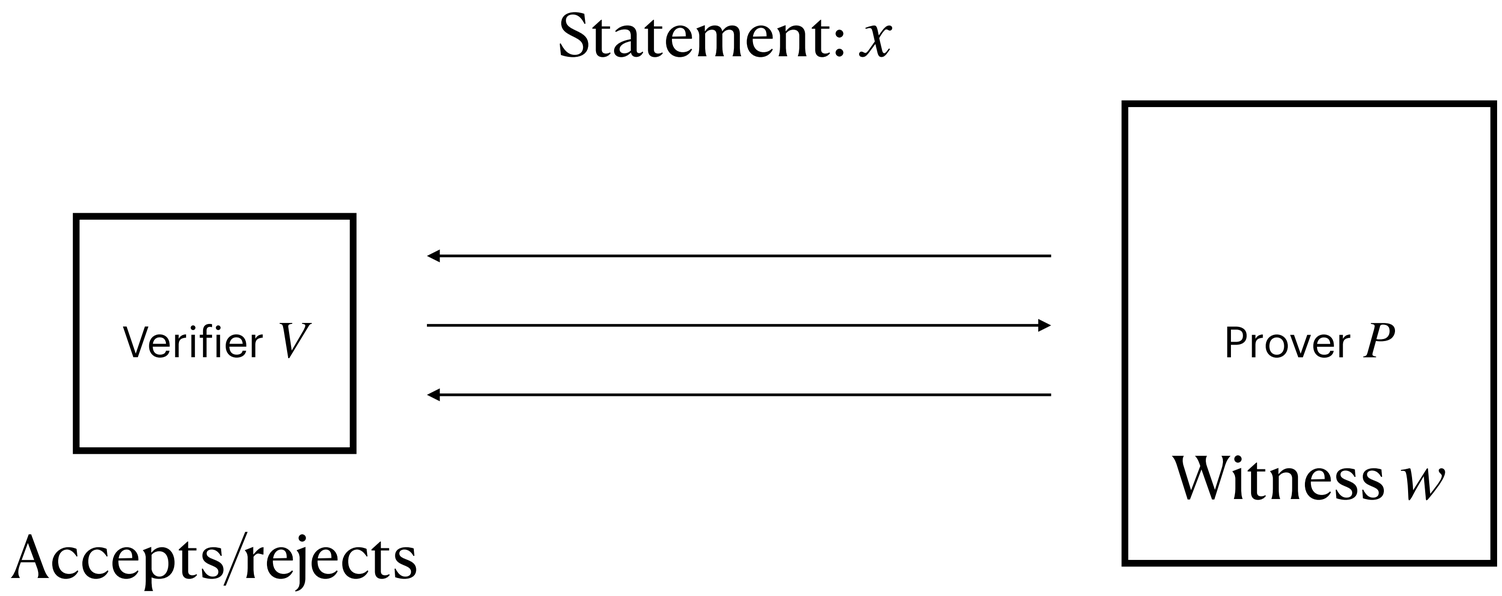
- **Completeness:** If $x \in \mathscr{L}$, then verifier accepts at the end of the interaction.

Zero-Knowledge Proofs



- **Completeness:** If $x \in \mathscr{L}$, then verifier accepts at the end of the interaction.
- **Soundness:** If $x \notin \mathscr{L}$, then verifier accepts with probability at most 1/3.

Zero-Knowledge Proofs



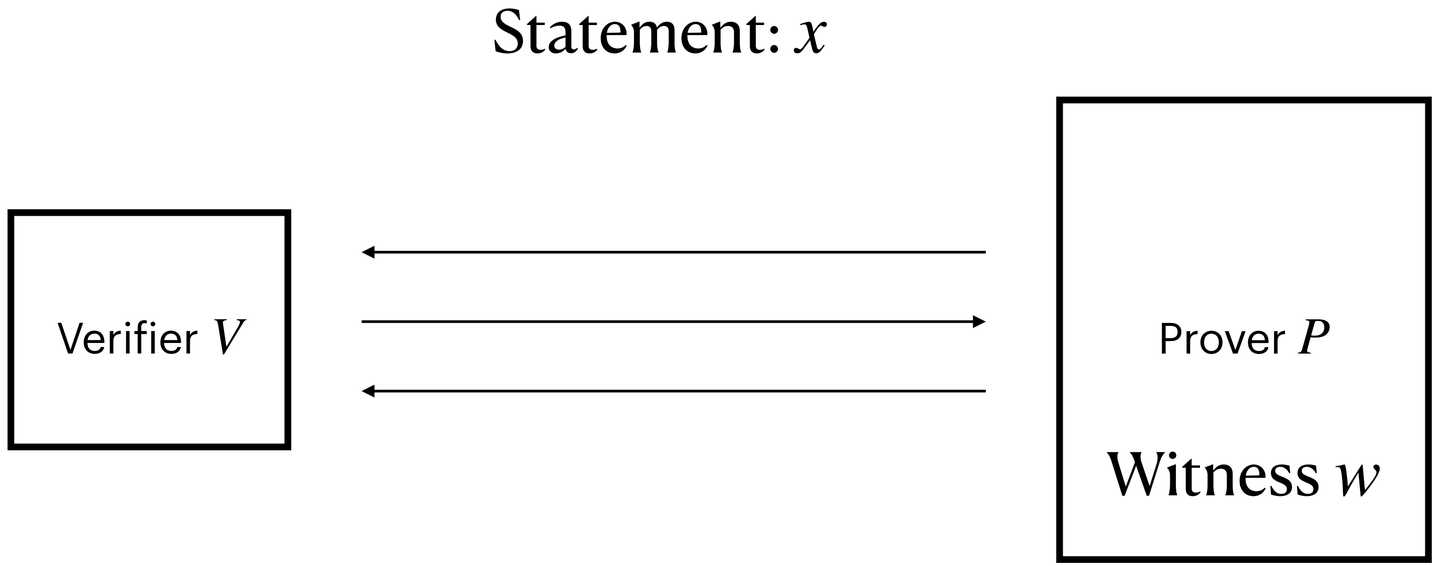
- **Completeness:** If $x \in \mathscr{L}$, then verifier accepts at the end of the interaction.
- **Soundness:** If $x \notin \mathscr{L}$, then verifier accepts with probability at most 1/3.
- **Zero-knowledge:** The verifier should learn no information about the witness.

How to formally define?

An Attempt

How to formally define?

An Attempt



How to formally define?

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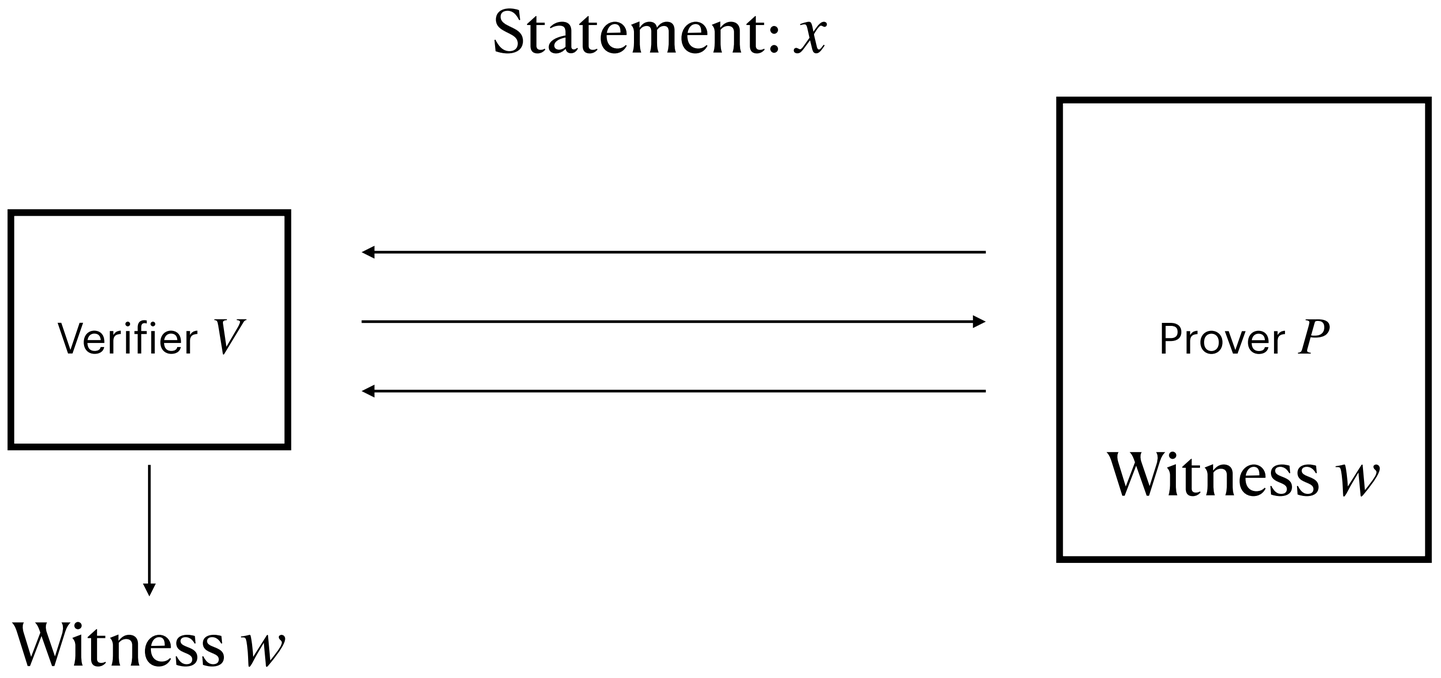

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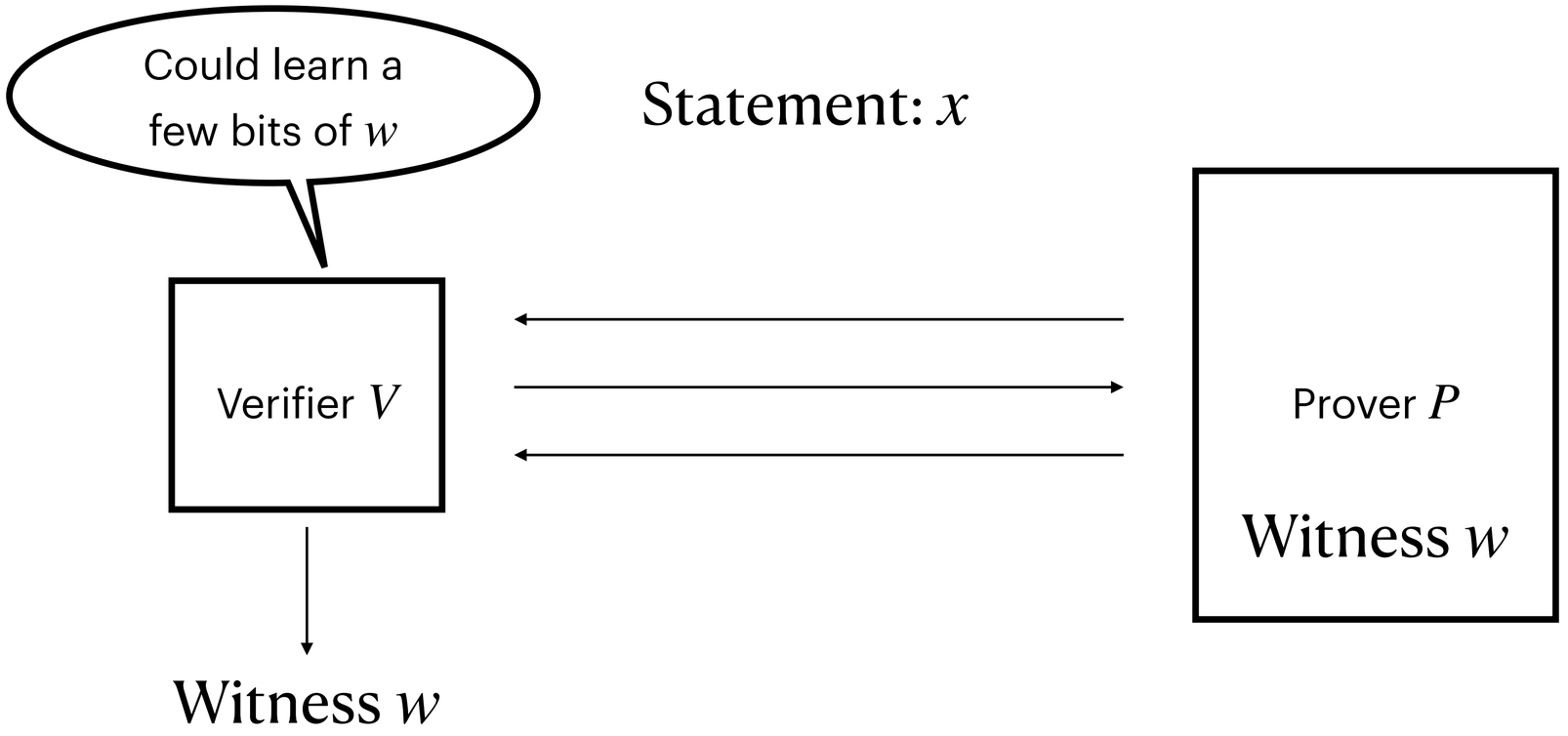

Formal Definition

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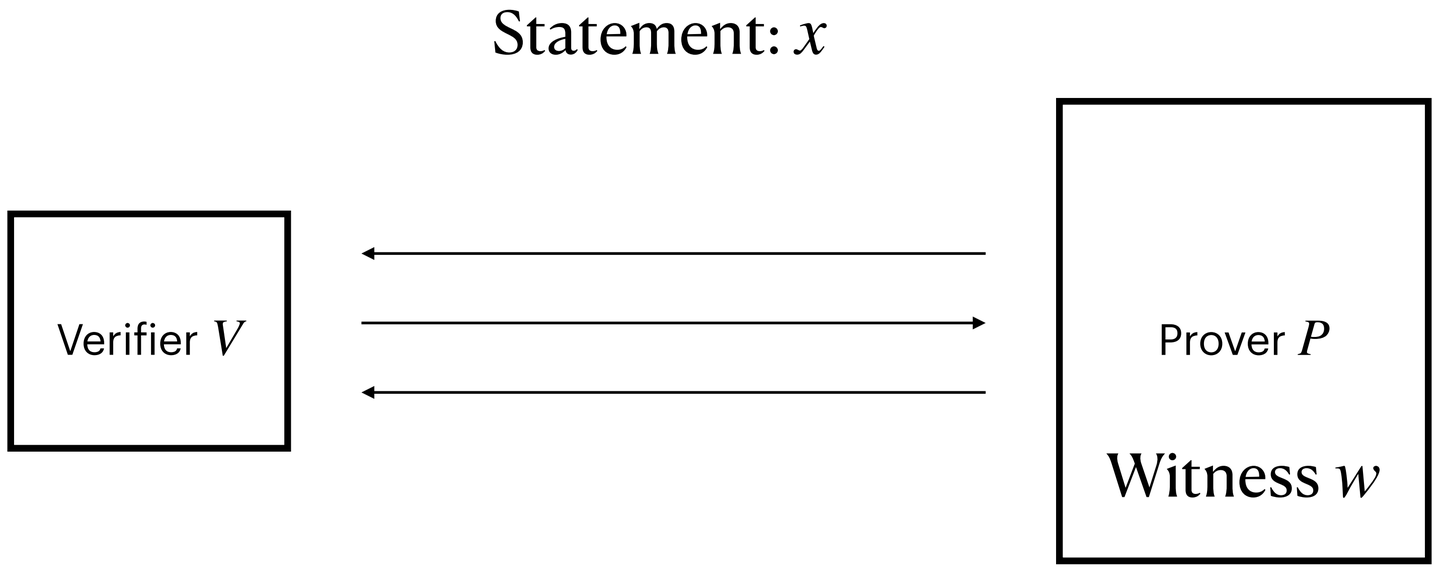

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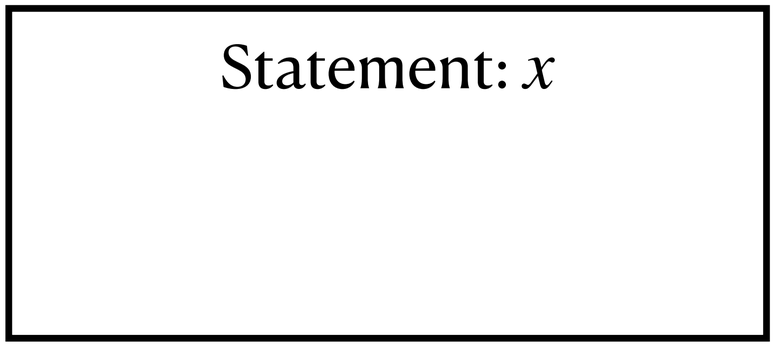

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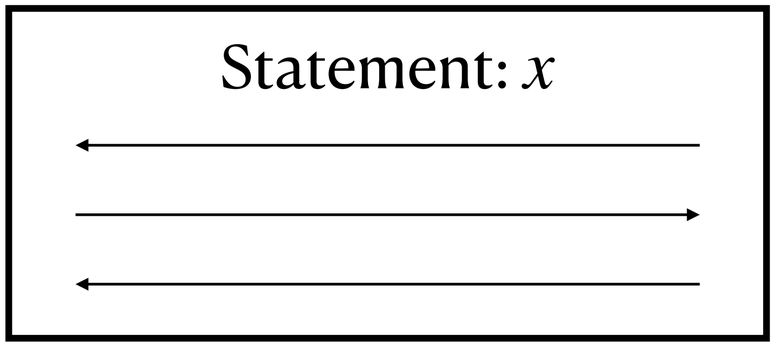

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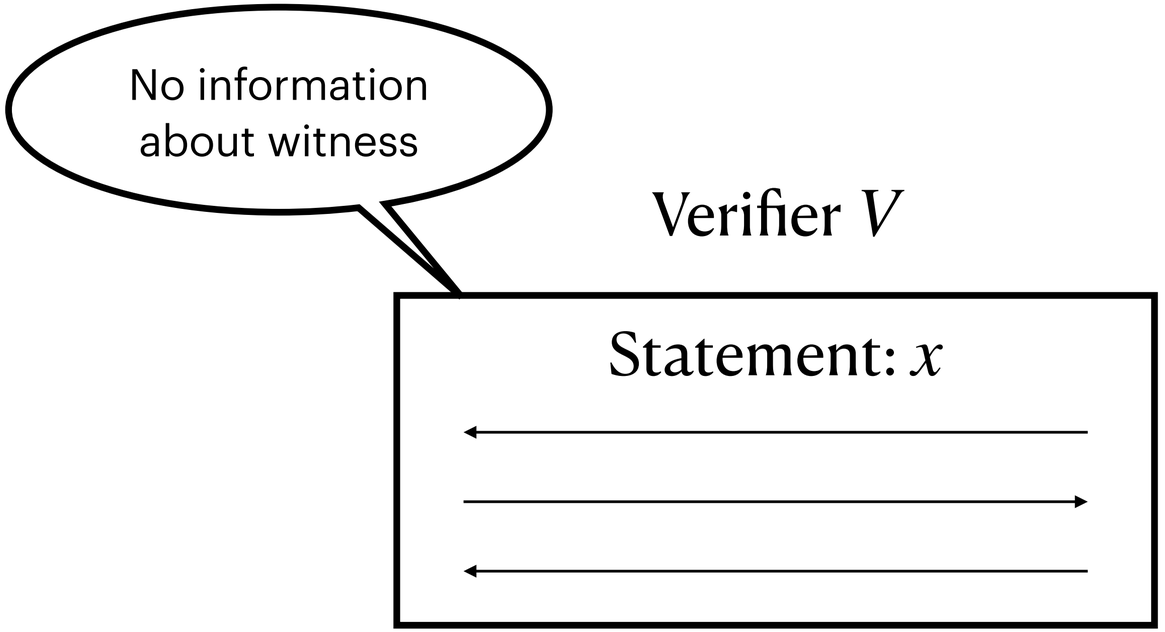

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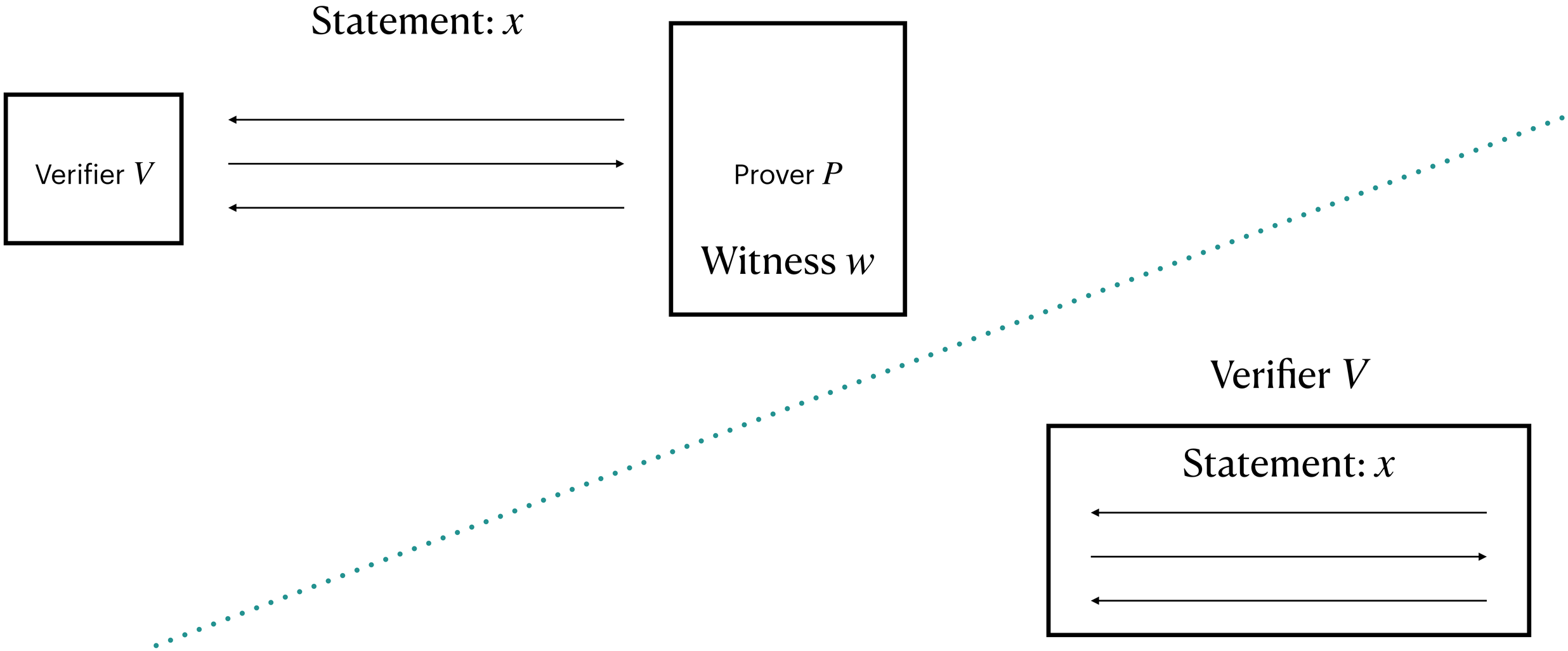

3-Colorability

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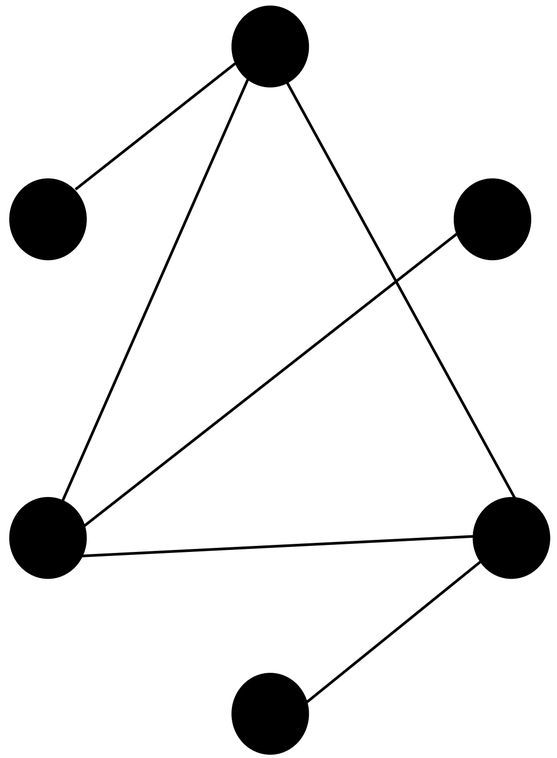

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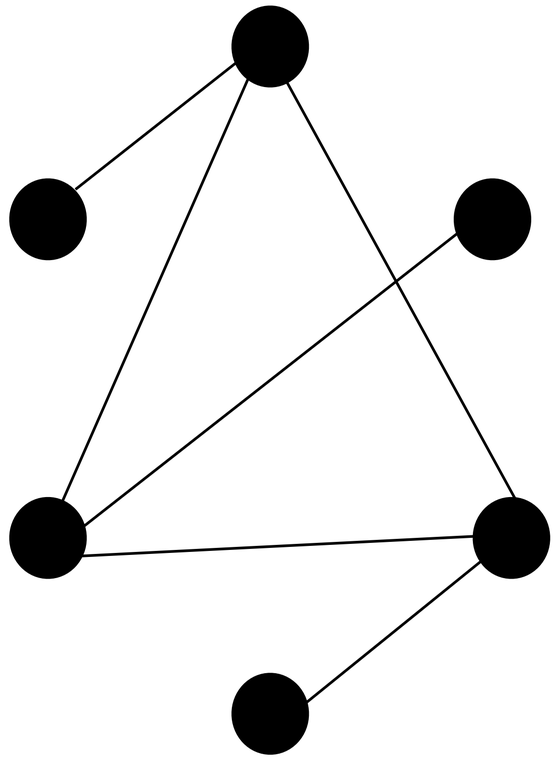

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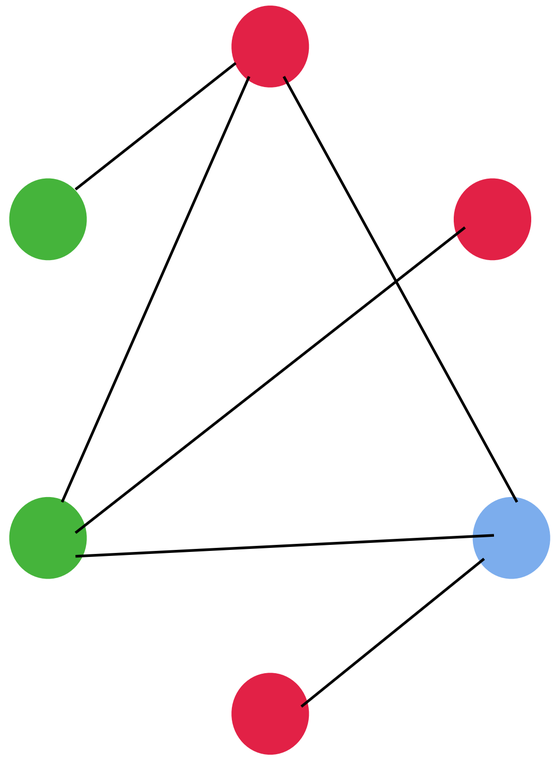
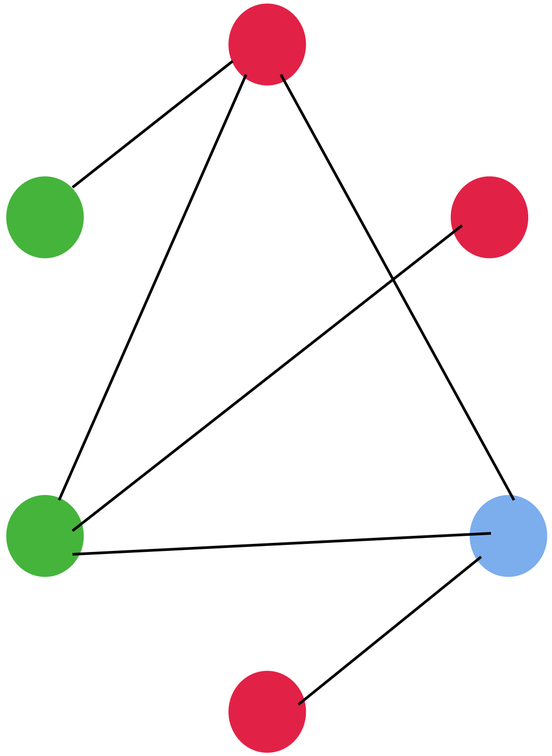


3-Colorability

3-colorability is NP-complete



Zero-knowledge proof for NP

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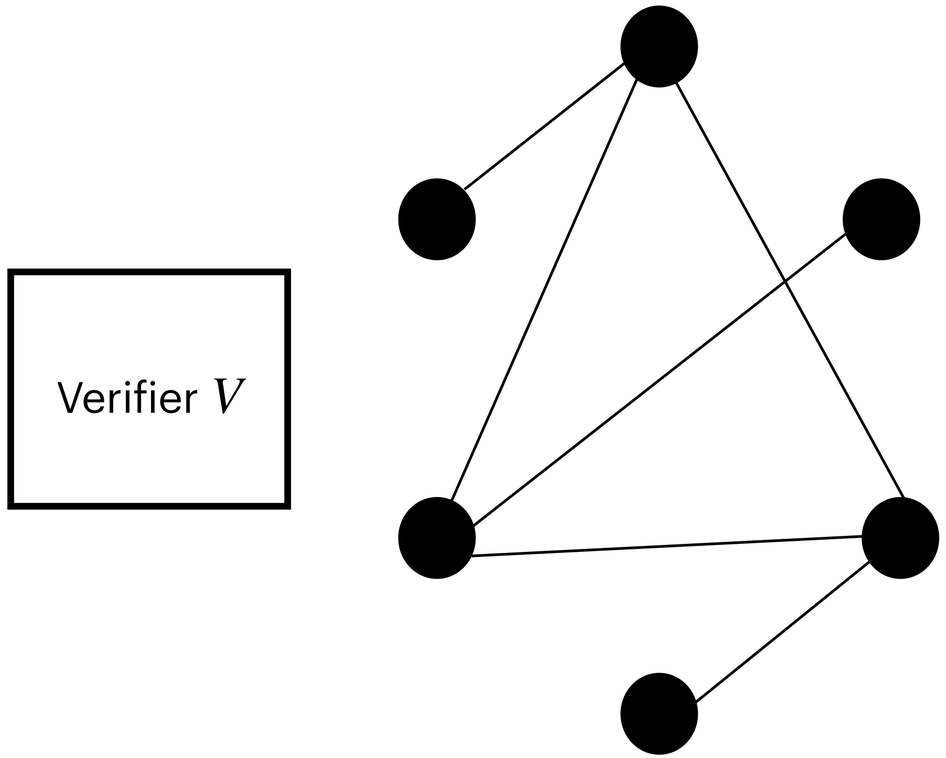

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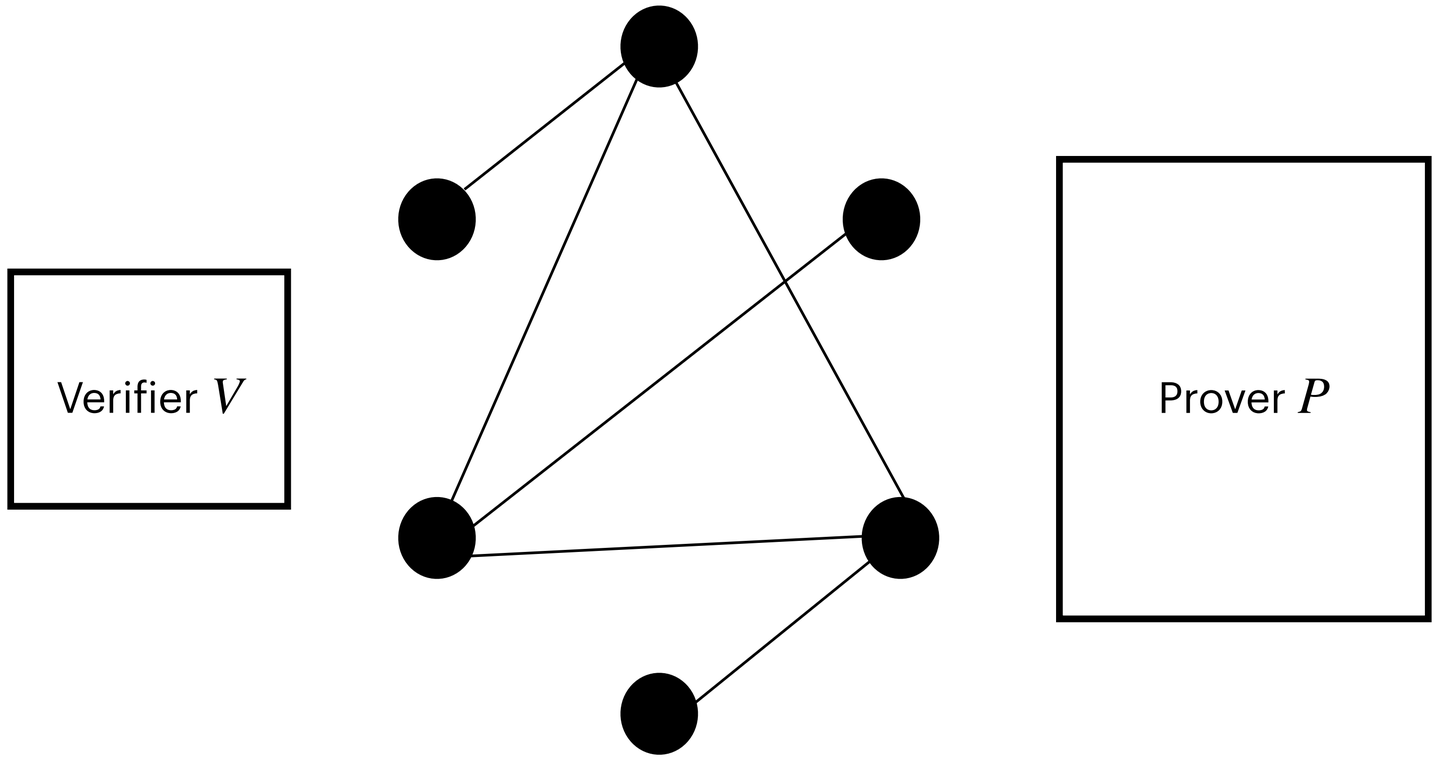

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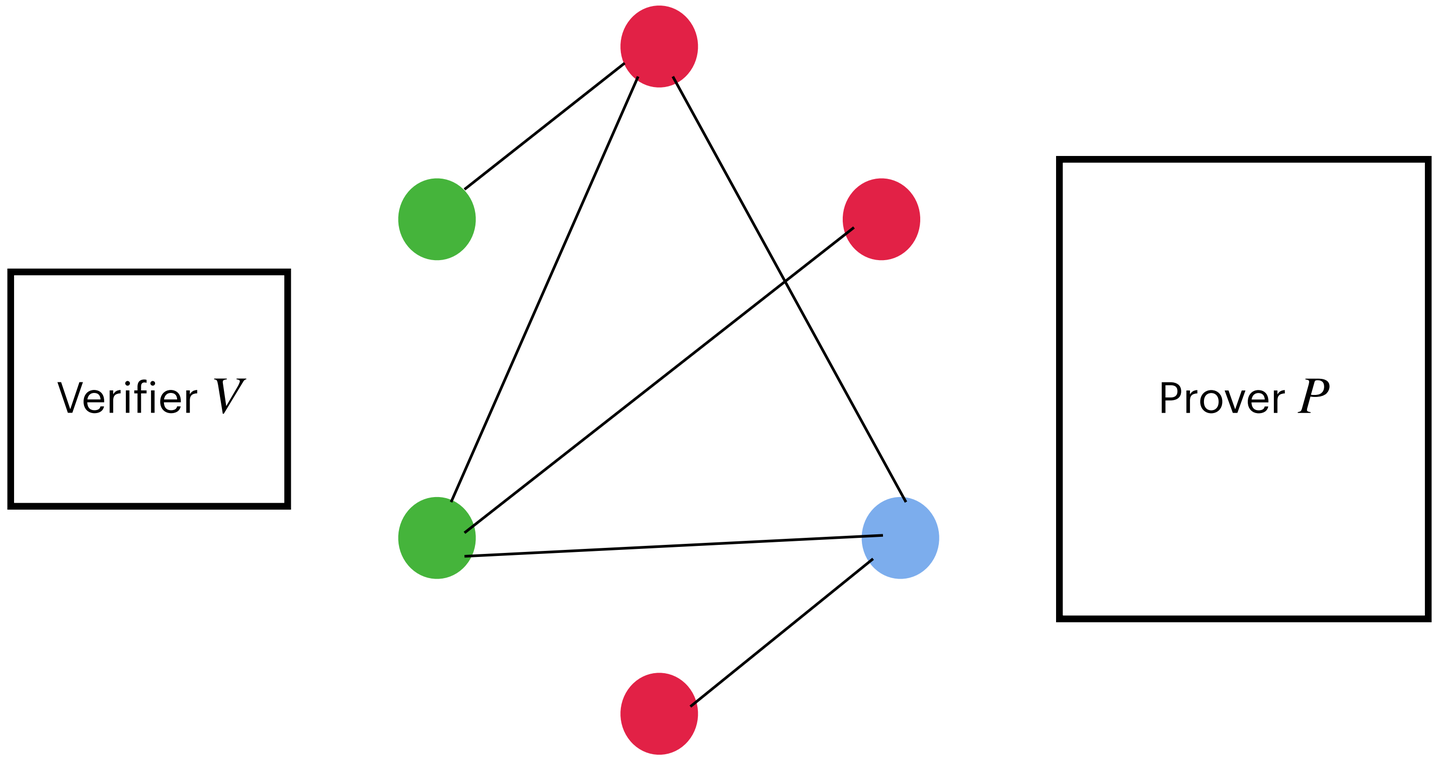

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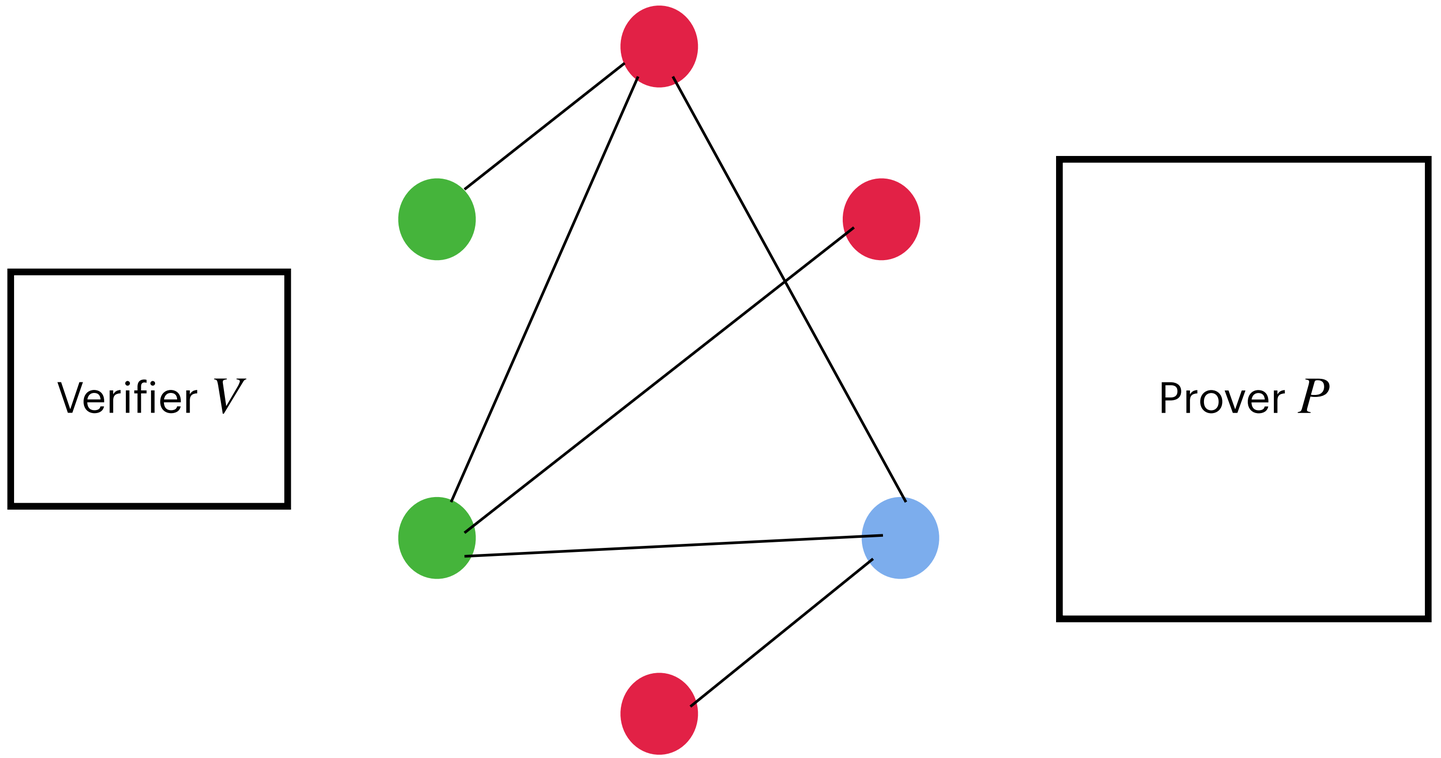
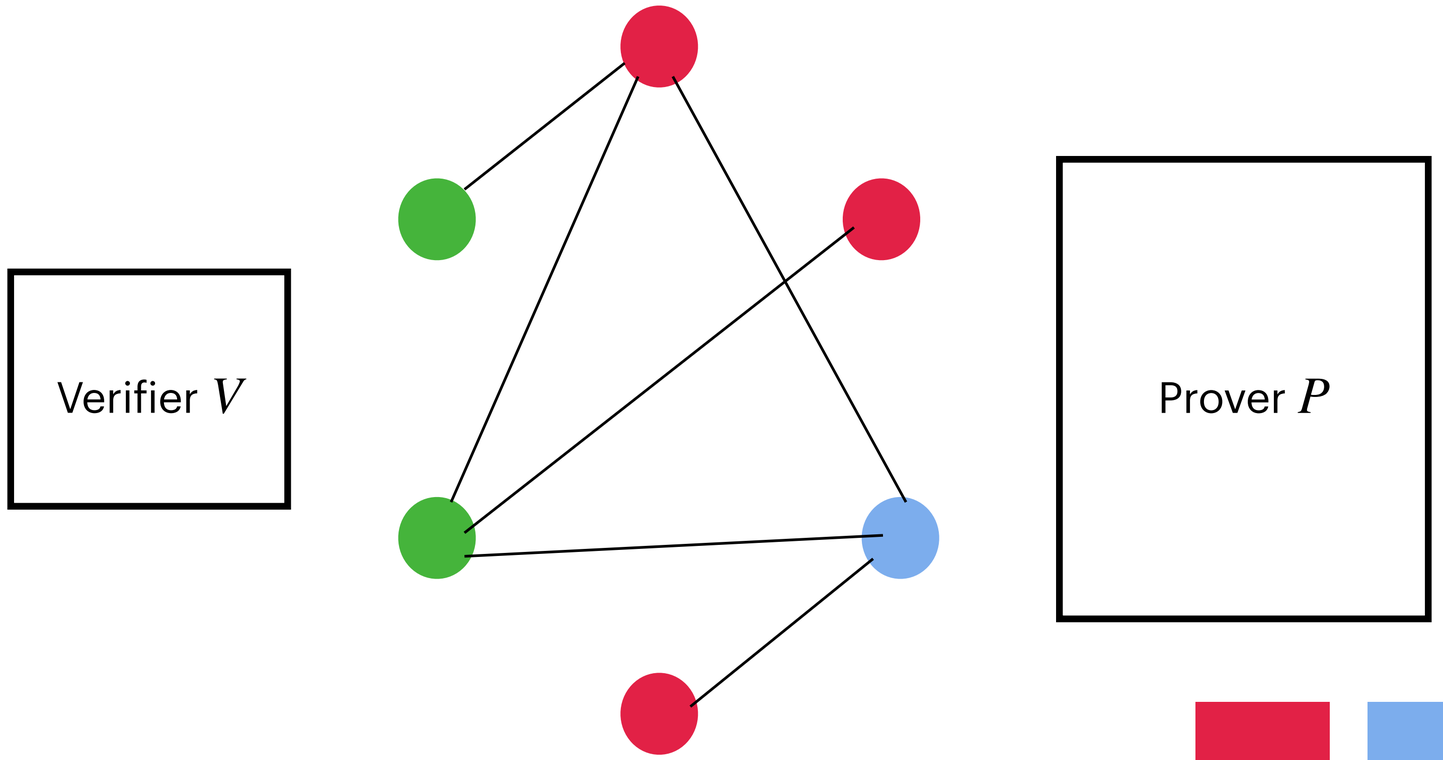
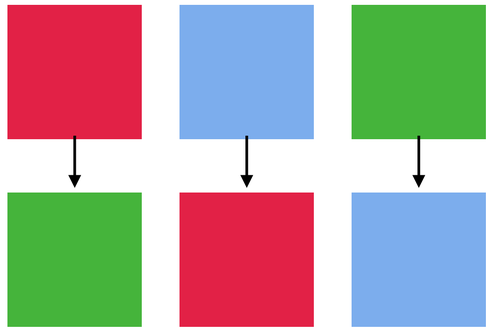


Step-1: Prover chooses a random permutation of the colors.

Zero-knowledge proof for NP



Step-1: Prover chooses a random permutation of the colors.



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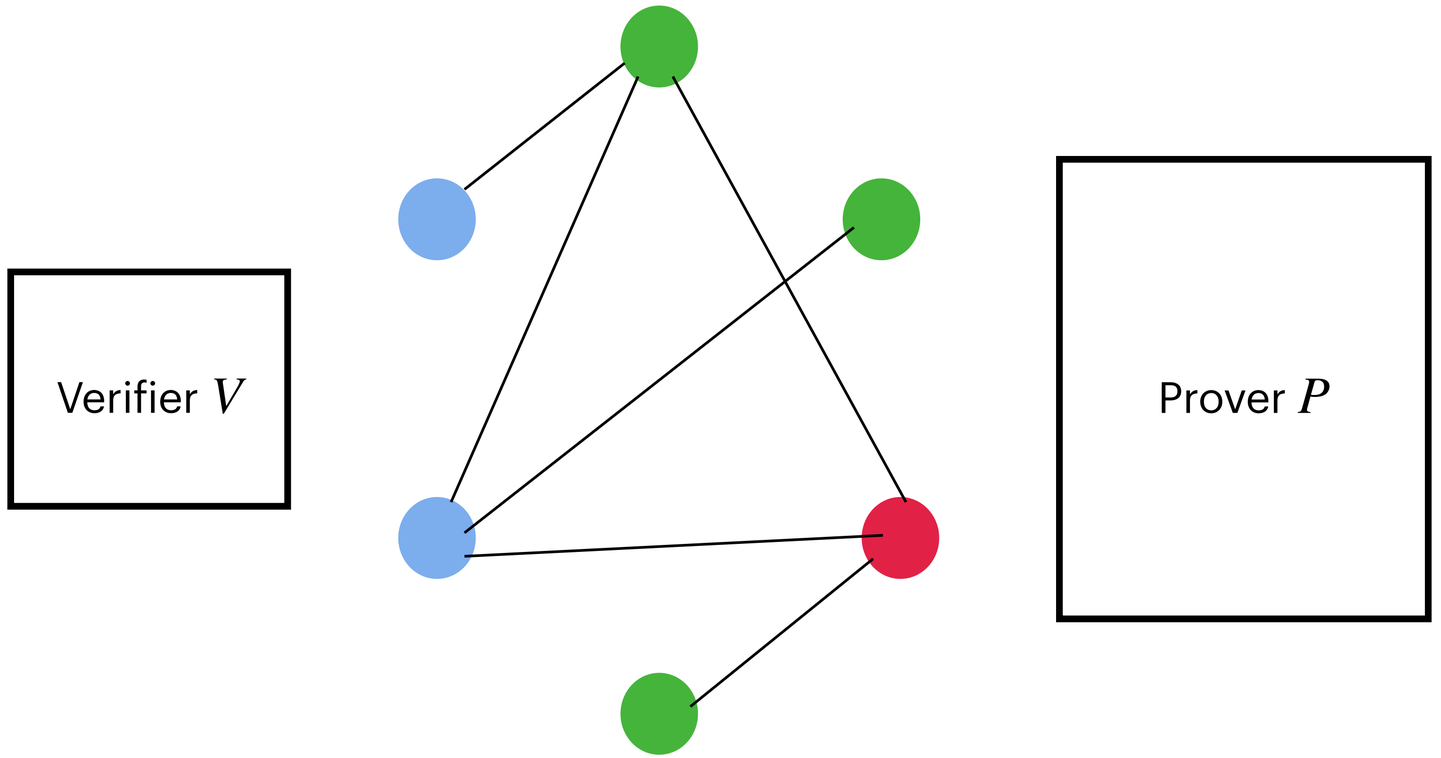

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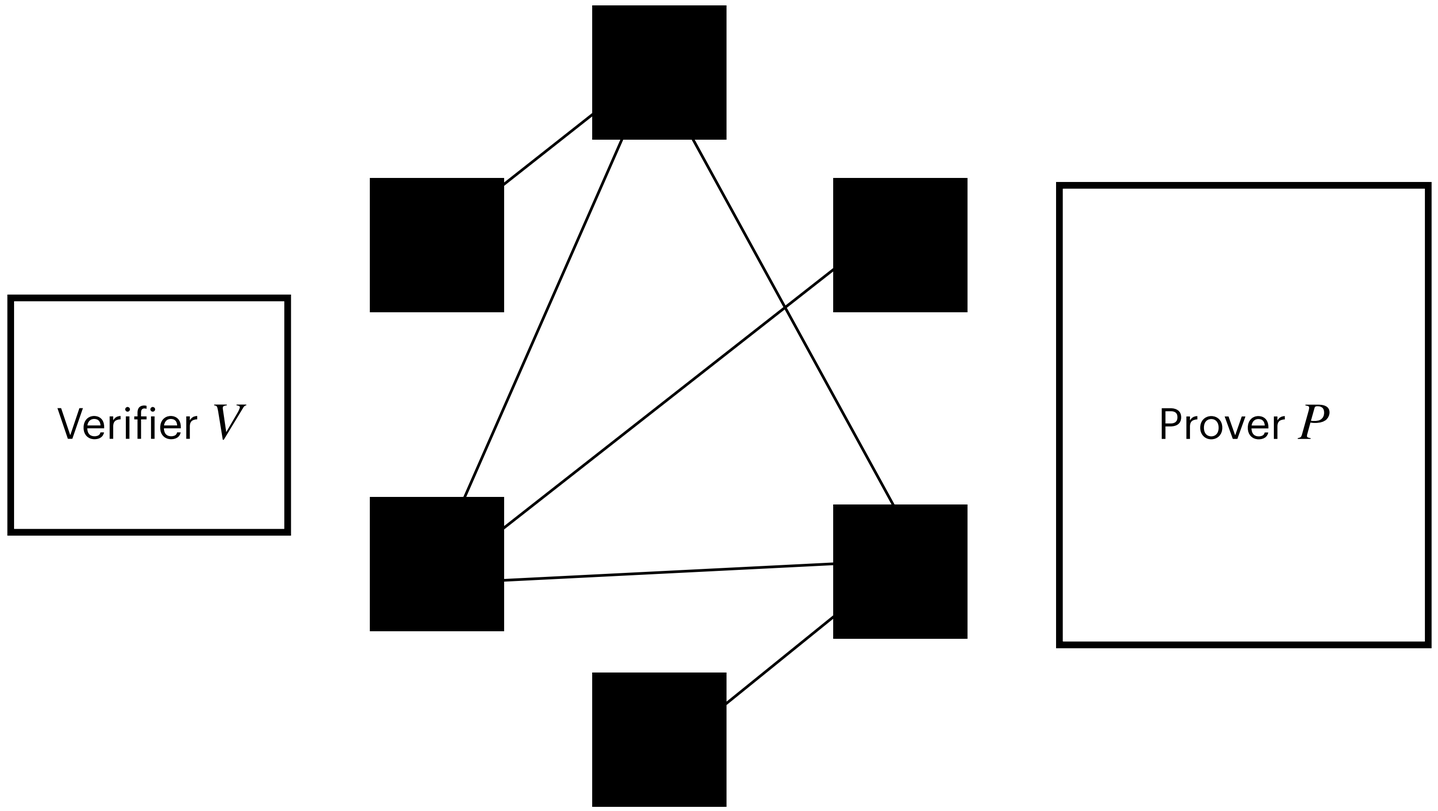
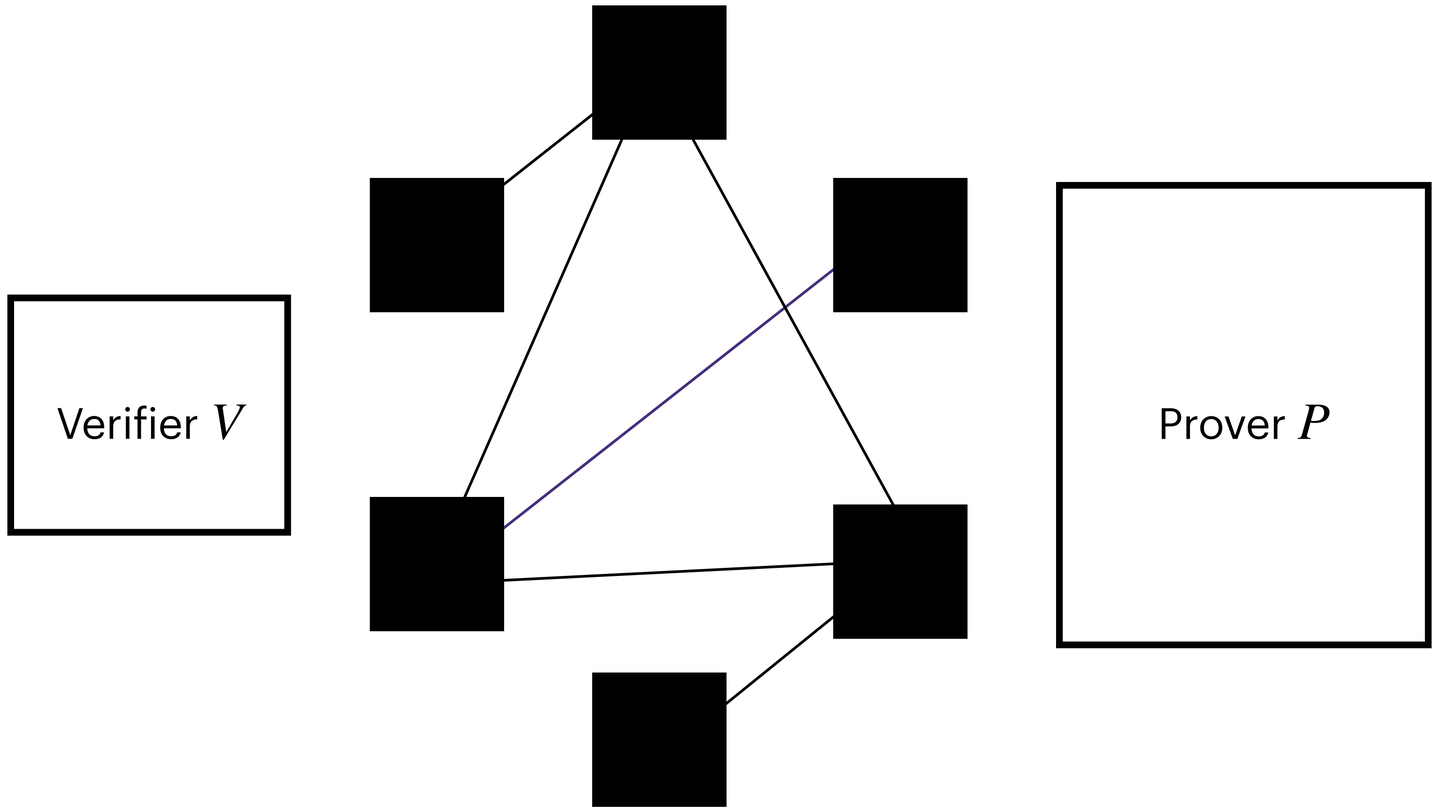


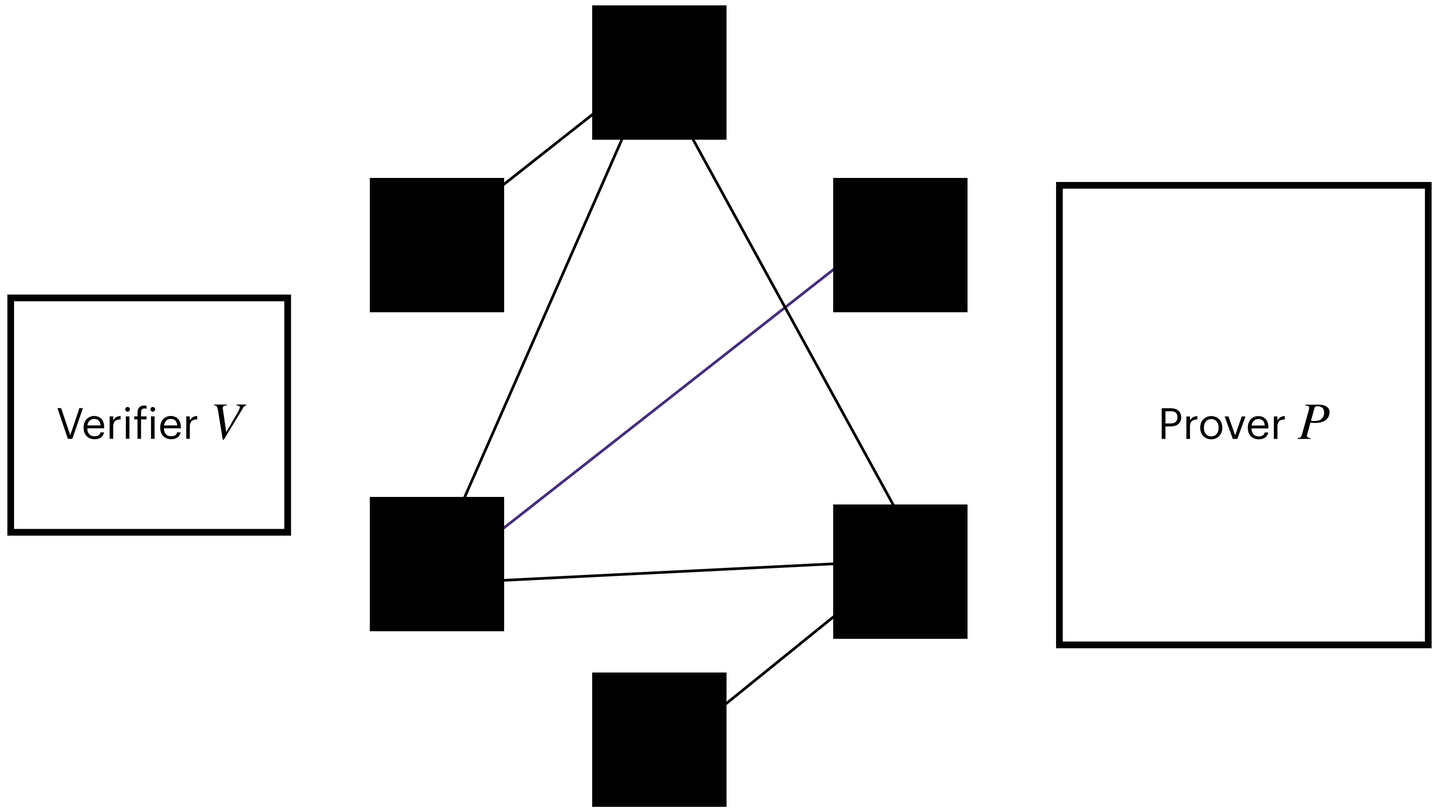
Step-2: Prover places each color behind a black-box

Zero-knowledge proof for NP



Step-3: Verifier chooses a random edge

Zero-knowledge proof for NP



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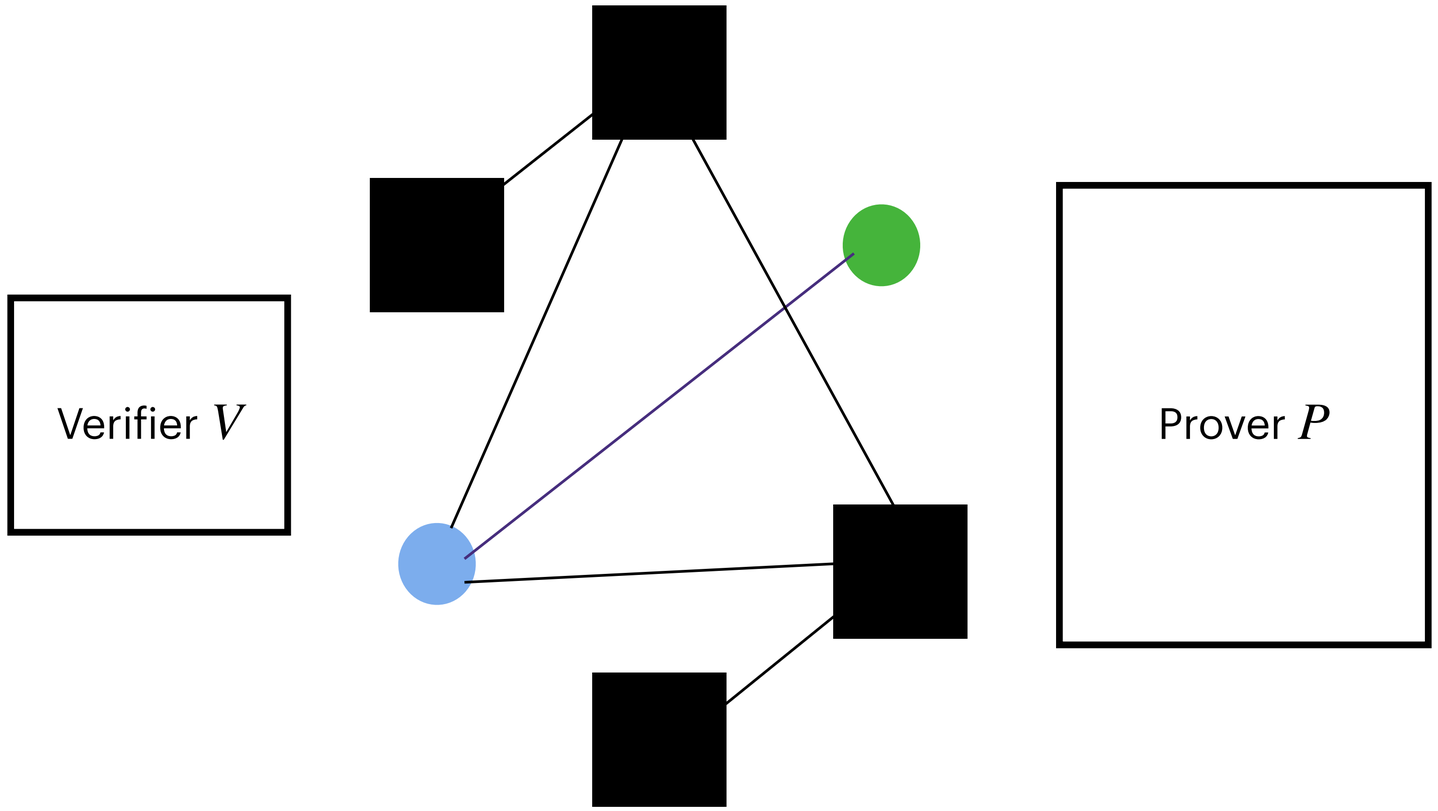
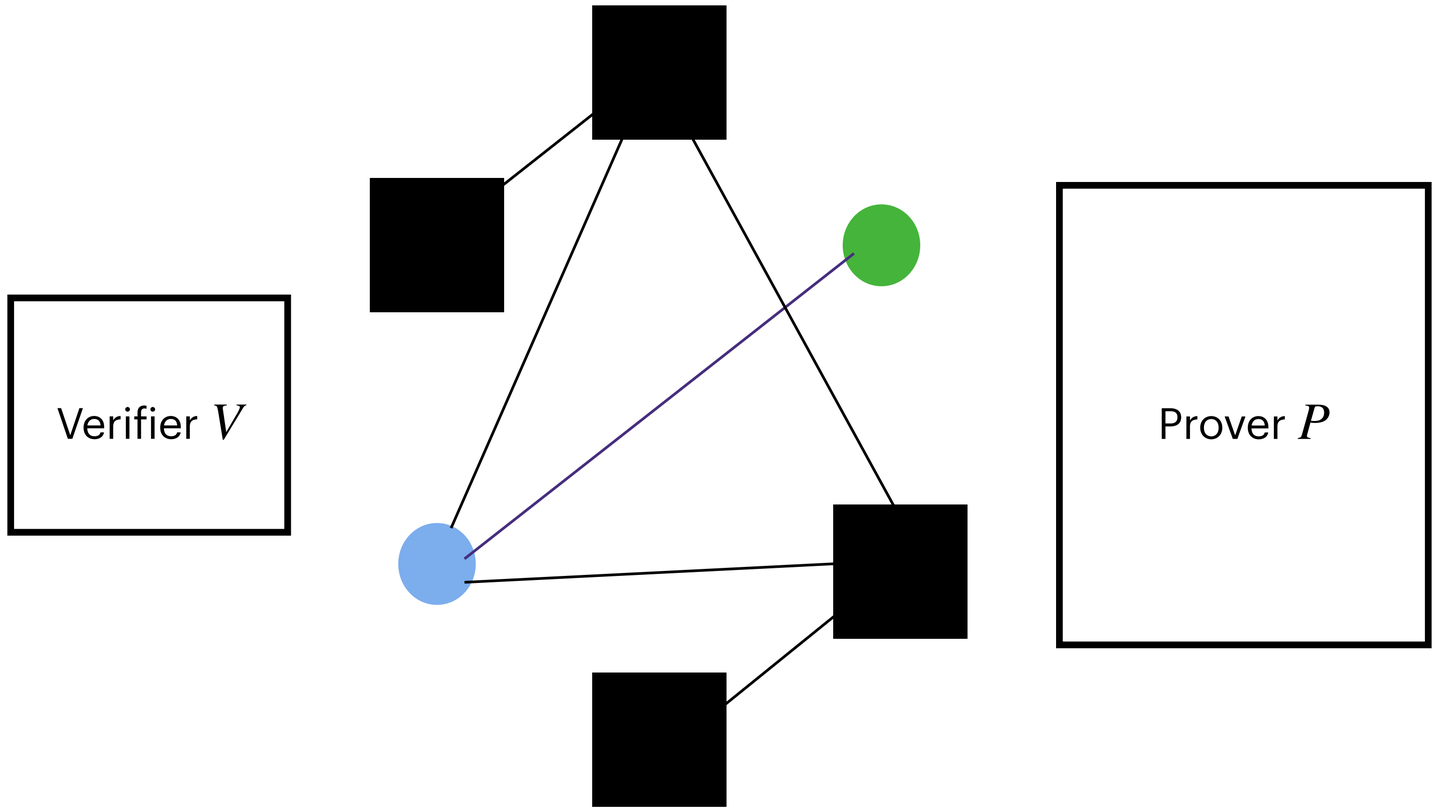


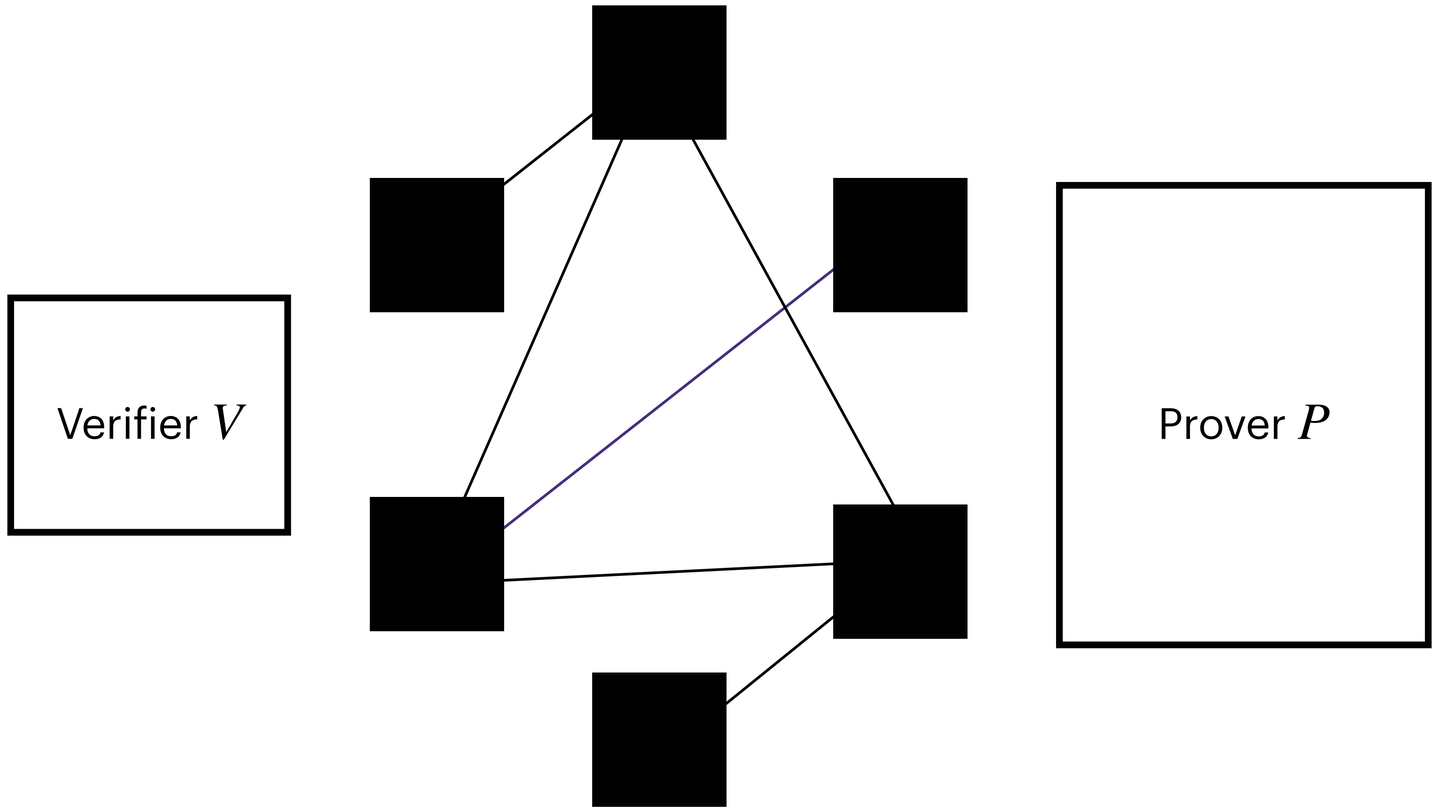
Step-4: Prover opens the colors on the endpoints of this edge.

Zero-knowledge proof for NP



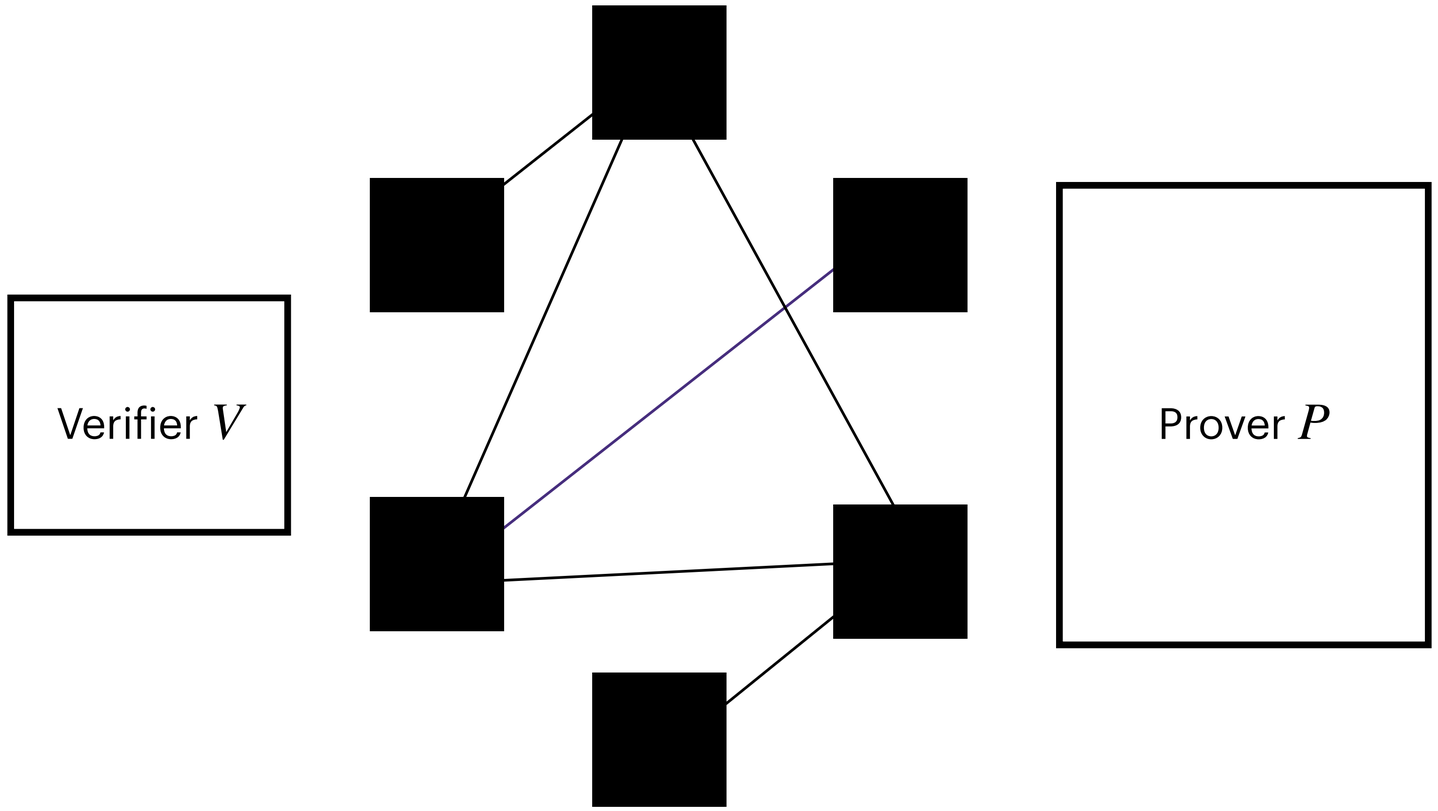
Step-5: Verifier checks if the endpoints are colored differently.

Completeness



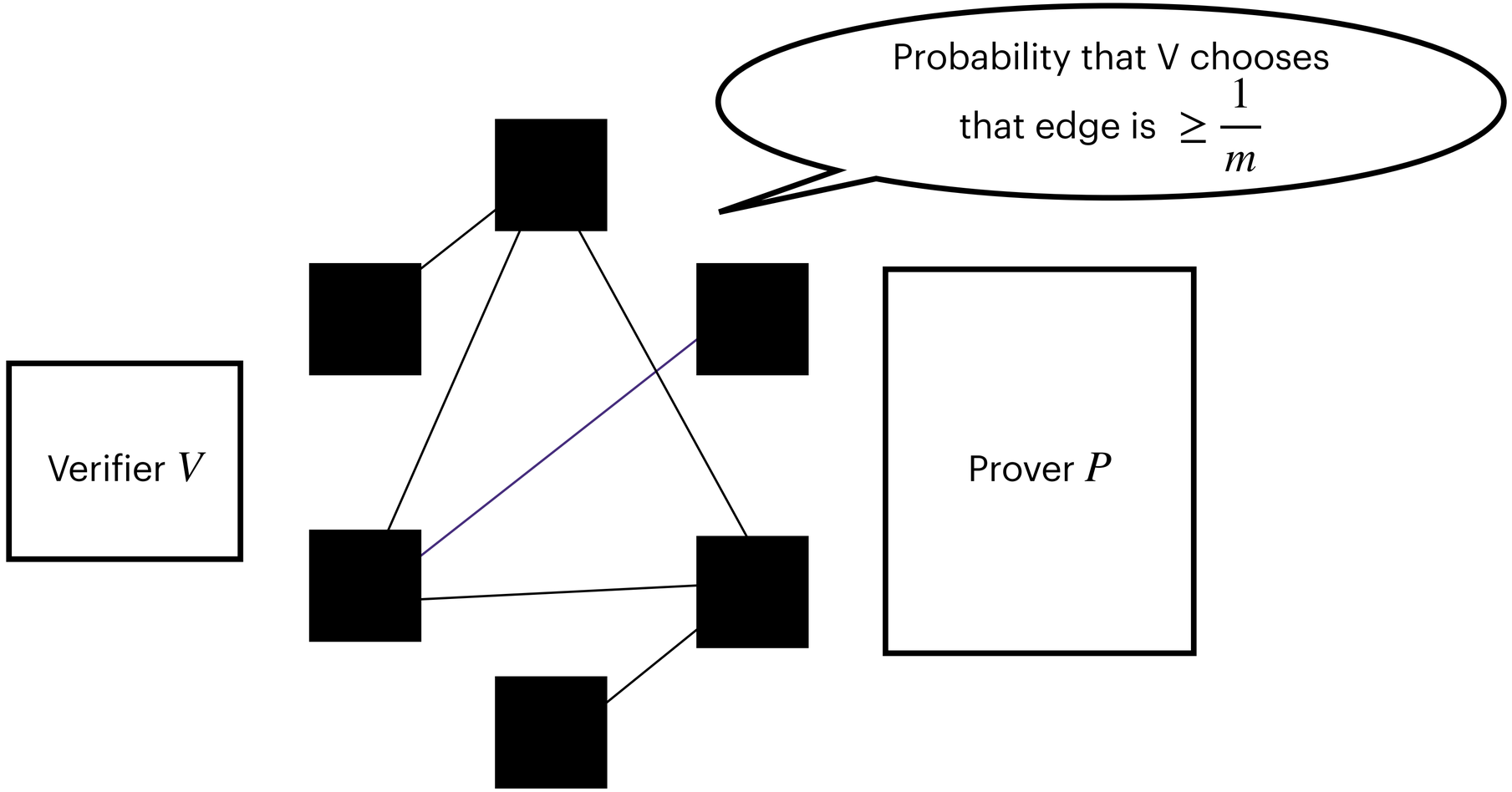
If the graph is 3-colorable, then the verifier always accepts.

Soundness



If the graph is not 3-colorable, then there exists at least one edge that has both endpoints colored using the same color.

Soundness



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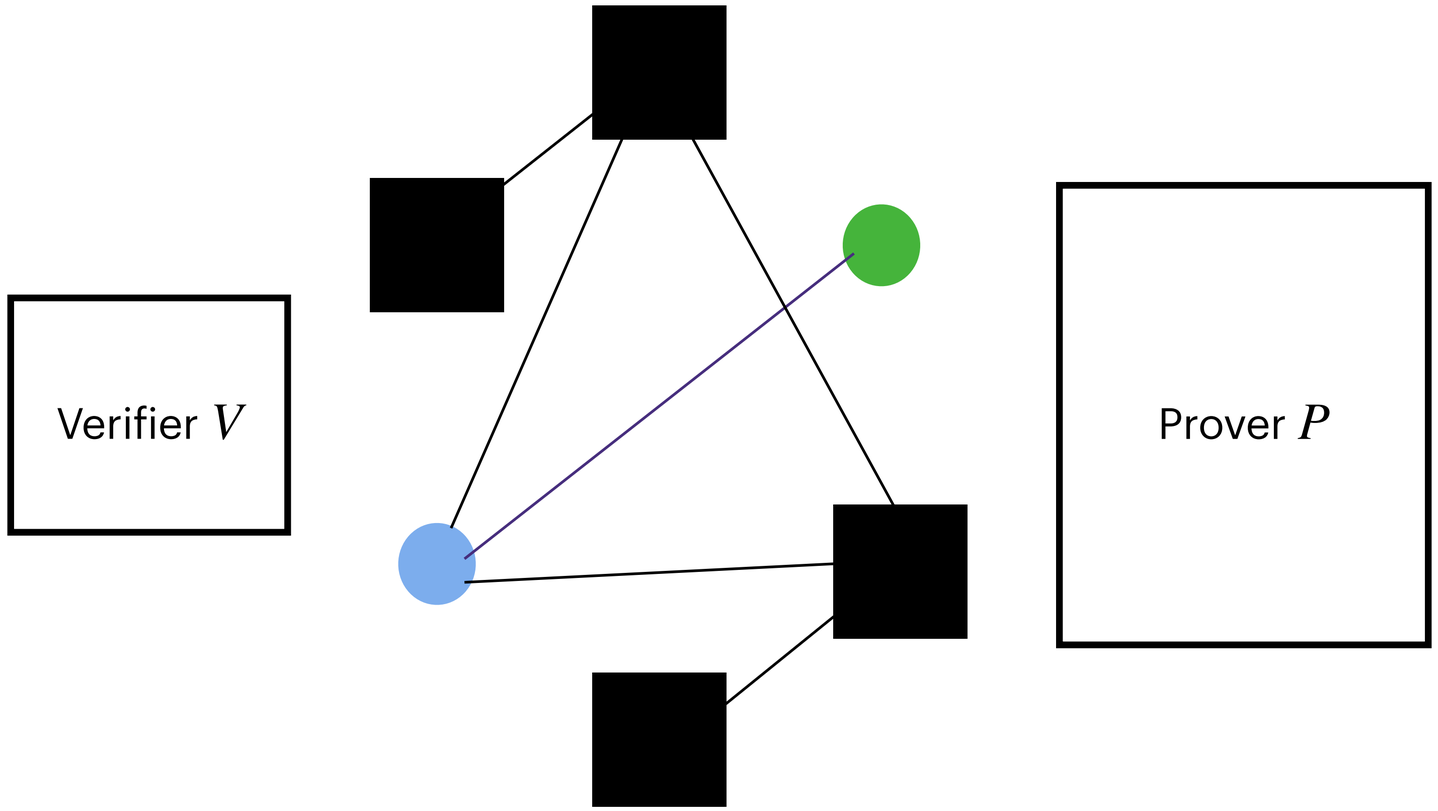
Zero-Knowledge



Zero-Knowledge