

**Due: Friday, December 1, 9AM EST**

This assignment is worth 15% of final grade. If you have no idea how to answer a question (or part of a question), you will receive 20% of the credit for that question (or subquestion) by stating “I do not know how to answer this question”. If your answer makes no sense, you will not receive any credit. Any answer that shows some understanding of the question will receive some credit.

1. (10 points) (Suppose tomorrow someone proves that  $P \neq NP$ .  
Would the following language be decidable in polynomial time?  
 $\{ \langle G \rangle \mid G \text{ can be coloured with 100 colours} \}$ . Here  $\langle G \rangle$  denotes the encoding of the graph as a string over some alphabet. Explain your answer.
2. Suppose tomorrow someone proves that  $P = NP$ .
  - (a) (5 points) Explain why it might not be possible to use RSA for public key encryption.
  - (b) Explain (5 points) why it might still be possible to use RSA for public key encryption.
3. (10 points) Show how to polynomial time reduce the problem of finding a 3 colouring of a graph (if one exists) to the decision problem of deciding if a graph is 3-colourable.  
Hint: a graph triangle is a set of three nodes  $(a, b, c)$  with edges between each pair of nodes. A triangle requires 3 colours. Call these colours red, blue, green. How could you make sure that a node  $v \notin \{a, b, c\}$  is coloured (for example) green?
4. Please consult the Week 11 course slides (slide 7) for definitions of different fairness conditions when allocating a divisible resource of set  $S$  of items to  $n$  agents who have utilities for outcomes..
  - (5 points) Amongst the fairness criteria in the slides, which one do you think intuitively best represents your sense of fairness? Briefly explain your answer.
  - (5 points) Assume  $\cup_i A_i = S$ . That is, the entire resource is allocated. Show that for additive valuations that envy-freeness implies proportionality. Hint: Show the contrapositive that “an allocation that is not proportional is not envy-free”.