CSC196 **Assignment 2** **Fall, 2023**

Due: October 27, 9AM EST

This assignment is worth 15% of final grade. If you have no idea how to answer a question (or part of a question), you will receive 20% of the credit for that question (or subquestion) by stating "I do not know how to answer this question". If your answer makes no sense, you will not receive any credit. Any answer that shows some understanding of the question will receive some credit.

1. Consider the following graph colouring problem
   A simple graph $G = (V, E)$ has a k-colouring is there exists a function $\chi : V \to \{1, 2, \ldots, k\}$ such that $\chi(u) \neq \chi(v)$ for all $(u, v) \in E$. The graph colouring decision problem is "Given a graph $G = (V, E)$ and an integer k, decide if G has a k-colouring. The graph colouring problem is an NP-complete problem for $k \geq 3$.
   Note: You do not need to know anything about NP-complete problems other than that it is widely believed that such a problem cannot be done efficiently.
   However, for certain classes of graphs this question can be answered efficiently.

   - (5 points) For a fixed k, roughly estimate the time it would take to decide if $G = (V, E)$ has a k-colouring if you wanted to naively try all possible functions $\chi : V \to \{1, 2, \ldots, k\}$. State your time estimate in terms of $n = |V|, m = |E|$, and k.
   - (5 points) Suppose $G = (V, E)$ is a tree. That is, G is connected and has no cycles. Show that G has a 2-colouring. Hint: start colouring the tree at the root.
   - (5 points) Is every tree a bipartite graph?
   - (5 points) Is every bipartite graph a tree?

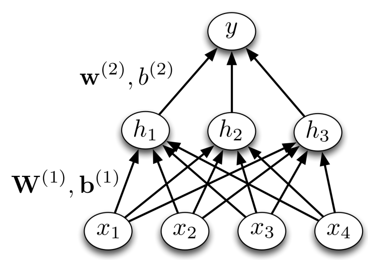
2. (10 points) As discussed in class, we know that the halting problem is undecidable. That is, given $(< M >, w)$ it is undecidable if M will halt on input w. Recall, we proved this result using a diagonalization argument.
   Suppose tomorrow that a new physical phenomena and computer architecture was discovered that solves the halting problem for Turing machines.

   Is there anything in Turing's work that might still be considered a great idea? Note: This is a thought question and one where there is not necessarily any best answer so the question will be graded on the plausability of your answer.

3. (10 points) In this question, you need to find a set of weights and biases for a neural net (with one hidden layer as below) for the following decision problem. Assume x_1, x_2, x_3, x_4 are integers satisfying $x_1 \leq x_2 \leq x_3 \leq x_4$. Compute the following function f:

$$y = f(x_1, x_2, x_3, x_4) = \begin{cases} 1 & \text{if } x_i \neq x_j \;\; \forall i \neq j. \\ 0 & \text{otherwise} \end{cases}$$

You will use the following architecture.



All of the hidden units and the output unit use a hard threshold activation function:

$$\phi(z) = \begin{cases} 1 & \text{if } z \geq 0 \\ 0 & \text{if } z < 0 \end{cases}$$

Provide a set of weights and biases for h_1, h_2, h_3 and y so that the network implements the function f.