Due: Wednesday, November 2, 8AM EST
NOTE change of due date!

This assignment is worth 15% of final grade. Each question is worth 20 points. If you have no idea how to answer a question (or part of a question), you will receive 20% of the credit for that question (or subquestion) by stating "I do not know how to answer this question". If your answer makes no sense, you will not receive any credit. Any answer that shows some understanding of the question will receive some credit.

1. Consider the following graph colouring problem (which we already encountered in Assignment 1):
   A simple graph $G = (V, E)$ has a k-colouring is there exists a function $\chi : V \rightarrow \{1, 2, \ldots, k\}$ such that $\chi(u) \neq \chi(v)$ for all $(u, v) \in E$. The graph colouring problem is "Given a graph $G = (V, E)$ and an integer k, decide if G has a k-colouring.

   - (5 points) For a fixed k, roughly estimate the time it would take to decide if $G = (V, E)$ has a k-colouring if you wanted to naively try all possible functions $\chi : V \rightarrow \{1, 2, \ldots, k\}$.
   - (5 points) Suppose $G = (V, E)$ is a tree. That is, G is connected and has no cycles. Show that G has a 2-colouring. Hint: start colouring the tree.
   - (5 points) For any k, show how to transform the k colouring problem to the $k + 1$ colouring problem; that is, transform a graph G to a graph G' such that G has a k-colouring iff and only if G' has a $k + 1$ colouring.

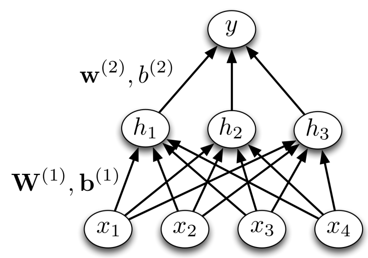
2. (10 points) As discussed in class, we know that the halting problem is undecidable. That is, given $(< M >, w)$ it is undecidable if M will halt on input w. Recall, we proved this result using a diagonalization argument.
   Suppose tomorrow that a new physical phenomena and computer architecture was discovered that solves the halting problem for Turing machines.

   Is there anything in Turing's work that might still be considered a great idea? Note: This is a thought question and one where there is not necessarily any best answer so the question will be graded on the plausability of your answer.

3. (10 points) In this question, you need to find a set of weights and biases for a neural net (with one hidden layer as below) for computing the following function f:
$y = f(x_1, x_2, x_3, x_4)$ is

$$f x_1, x_2, x_3, x_4) = \begin{cases} 1 & \text{if } x_1 < x_2 < x_3 < x_4 \\ 0 & \text{otherwise} \end{cases}$$

You may assume that the x_i are distinct rational numbers; i.e., $x_i \neq x_j$ for $i \neq j$. You will use the following architecture.



All of the hidden units and the output unit use a hard threshold activation function:

$$\phi(z) = \begin{cases} 1 & \text{if } z \geq 0 \\ 0 & \text{if } z < 0 \end{cases}$$

Provide a set of weights and biases for h_1, h_2, h_3 and y so that the network implements the function f.