Due: Friday, October 7, 8AM EST

This assignment is worth 15% of final grade. Each question is worth 20 points. If you have no idea how to answer a question (or part of a question), you will receive 20% of the credit for that question (or subquestion) by leaving the question (or subquestion) blank. If your answer makes no sense, you will not receive any credit. Any answer that shows some understadning of the question will receive some credit.

1. This exercise pertains to the simplified floating point representations (where we use a sign bit for the sign of the exponent) as described in the class and on slide 15 of the Week 2 slides. Assume each such number is 8 bits with one bit for the sign of the number, 4 bits for the exponent and 3 bits for the significand.

   - (10 points) Show how to represent the decimal number 15 as a simplified 8 bit floating point number.
   - (10 points)
     Explain why the decimal integer 17 *cannot* be represented exactly by such an 8 bit simplified floating point number.

2. Consider a sorted list in an array and a perfectly balanced search tree for searching in a dictionary having n identifiers/keys. What is the asymptotic time (e.g. $O(1)$ = constant time, $O(\log n), O(n)$) required to answer the following queries:

   - (10 points) What is the largest value key in the dictionary?
   - (10 points) What is the median value key is the dictionary? For simplicity let n be an odd integer so that the median value is well defined.

   You need to answer this for both data structures so that there should be four answers.

3. (20 points) The graph coloring (or colouring if we follow British/Canadian spelling) problem is the following. Given a graph (meaning an undirected graph) $G = (V, E)$, a valid k-colouring is a map $\chi : V \to \{1, 2, \ldots k\}$ such that $(u, v) \in E$ implies $\chi(u) \neq \chi(v)$. We think of these integers as k colours. We note that for $k \geq 3$, it is known that it is an NP-complete problem to determine if G has a valid k-colouring. **Note:** You do not need to know anything about NP-complete problems other than that it is widely believed that such a problem cannot be done efficiently.

   However, for certain classes of graphs this question can be answered efficiently. Consider a bipartite graph $G = (V, E)$ where $V = U_1 \cup U_2$ and $U_1 \cap U_2 = \emptyset$ and $E = U_1 \times U_2$; that is, U_1, U_2 is a partition of the nodes and all edges (u, v) are of the form $u \in U_1, v \in U_2$. It should be clear that every bipartite graph has a 2-colouring.

   You now need to argue the converse statement; namely, if $G = (U, V)$ has a valid 2-colouring, then G is bipartite. In effect I am asking for a proof of this statement but don't be bothered by the word "proof". Give what you believe is a convincing argument.