

**Due: Friday, December 3, 8AM EST**

This assignment is worth 15% of final grade. Each question is worth 10 points. If you have no idea how to answer a question (or part of a question), you will receive 20% of the credit for that question (or subquestion) leaving the question (or subquestion) blank. If your answer makes no sense, you will not receive any credit. Any answer that shows some understanding of the question will receive some credit.

1. 10 pts • Let  $A \subset \Sigma^*$  and  $B \subset \Sigma^*$  be languages (decision problems). Define  $A \cap B$  to be  $\{w \mid w \in A \text{ and } w \in B\}$ .

Explain the following fact: If  $A$  and  $B$  are  $NP$  sets, then  $A \cap B$  is also an  $NP$  set.

- Show that there is a polynomial time transformation that transforms an instance of  $(k+1)SAT$  to an instance of  $kSAT$  for any  $k \geq 3$ .  
Hint:  $z \equiv x \vee y$  can be represented by a 3CNF formula.  
Why won't this idea work for  $k = 2$ ?

2. 5 pts Show that  $P = NP$  would imply there cannot be any one-way functions. See definition of one-way function on slide 28 of Week 10 slides.

3. 15 pts Consider a large social network of friends. That is, assume we have an undirected network  $G = (V, E)$  where the nodes in  $V$  are people and an edge  $(u, v)$  means that  $u$  and  $v$  are friends. The nodes  $v \in V$  have weights  $w_v$  reflecting the importance of node  $v$  and the edges  $(u, v)$  have weights  $\eta_{(u,v)}$  reflecting the strength of that friendship.

The following are thought questions.

- Using the terminology and examples in the W11 slides (e.g., slides 17,19,22,22,23) for graphs/networks, and your own experience, how would you define a “community” of friends in the social network?
- Using your own experience, do you think that the generation of a social network is reasonably modeled by preferential attachment. See slides 37-41 in the W11 slides.
- If you had a small amount of money (or other incentives) to influence a small number of people in the network (and hope that those people would in turn influence a targeted community), how would you decide on which people to initially influence? Assume you can randomly access nodes in the network and any “local” graph properties of those nodes (e.g., degree, weight, correlation coefficient, weight of adjacent edges), and similar local graph information about adjacent nodes, as well as any stated content information for a node and its adjacent nodes (e.g., occupation, age, address).