Due: Wednesday, November 18, 11AM

This assignment is worth 15% of your final grade

1. Let $A(G)$ be the adjacency matrix of a simple (i.e,, no self loops and no parallel edges) directed graph $G = (V, E)$ and let $B(G) = A(G) + I$ where I is the identity matrix. That is, $B(G)$ represent the graph G with the addition of a self loop for each node. Consider the matrices A^k and B^k where $A^1 = A, B^1 = B, A^k = A * A^{k-1}$, and $B^k = B * B^{k-1}$ where $*$ denotes matrix multiplication. Let $A^k[i, j]$ and $B^k[i, j]$ (respectively) denote the i, j entry in the matrices A^k and B^k. For each of the questions below, try to use graph theory terminology.

   - In words, what is the meaning of each of the following:
     1) $A^k[i, j] = 0,\ 2) A^k[i, j] > 0,\ 3) B^k[i, j] = 0$ and 4) $B^k[i, j] > 0$?
     Hint: There is a path from node i to node j of length exactly (respectively, at most) k if and only if there some node node ℓ such that there is an edge (ℓ, j) and there is a path of length exactly (respectively, at most) length $k - 1$.
   - Let $|V| = n$; that is, $G = (V, E)$ has n vertices. Looking at all the entries $B^{n-1}[i.j]$ what can you say about G if $B^[i, j] > 0$ for all i, j?

2. Consider a large social network of friends. That is, we have an undirected network $G = (V, E)$ where the nodes in V are people and an edge (u, v) means that u and v are friends. The nodes $v \in V$ have weights w_v reflecting the importance of node v and the edges (u, v) have weights $\eta_{(u,v)}$ reflecting the strength of that friendship.

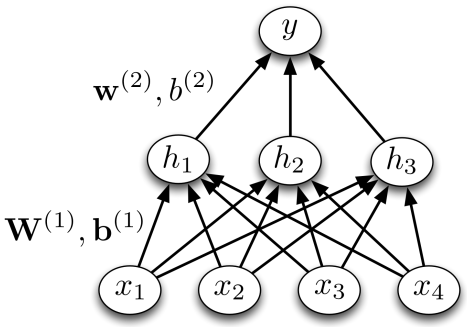
   The following are thought questions.

   - Using terminology from graph theory, how would you define a "community" of friends?
   - If you had a small amount of money (or other incentives) to influence a small number of people in the network (and hope that those people would in turn influecne a targetted community), how would you decide on which people to initially influence?

3. In this question, you need to find a set of weights and biases for a neural net (with one hidden layer as below) for computing the following function f:
$y = f(x_1, x_2, x_3, x_4)$ is

$$fx_1, x_2, x_3, x_4) = \begin{cases} 1 & \text{if } x_1 < x_2 < x_3 < x_4 \\ 0 & \text{otherwise} \end{cases}$$

You may assume that the x_i are distinct rational numbers; i.e., $x_i \neq x_j$ for $i \neq j$. You will use the following architecture.



All of the hidden units and the output unit use a hard threshold activation function:

$$\phi(z) = \begin{cases} 1 & \text{if } z \geq 0 \\ 0 & \text{if } z < 0 \end{cases}$$

Provide a set of weights and biases for h_1, h_2, h_3 and y so that the network implements the function f.