Today

- Ensemble Methods
- Bagging
- Boosting
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- Typical application: classification

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2. Each votes on test instance
3. Take majority as classification

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Aim: take simple mediocre algorithm and transform it into a super classifier without requiring any fancy new algorithm.
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Notes:
- Also known as meta-learning
- Typically applied to weak models, such as decision stumps (single-node decision trees), or linear classifiers
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Variance-bias Tradeoff

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- **Variance-bias decomposition** is a way of analyzing the generalization error as a sum of 3 terms: variance, bias and irreducible error (resulting from the problem itself)
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2. **Bias reduction**: for simple models, average of models has much greater capacity than single model (e.g., hyperplane classifiers, Gaussian densities).
   - Averaging models can reduce bias substantially by increasing capacity, and control variance by fitting one component at a time (e.g., boosting)
Ensemble Methods: Justification

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  - Accurate (better than guessing)
  - Diverse (different errors on new examples)
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Probability that majority vote wrong: error under distribution where more than $N/2$ wrong
Figure: Example: The probability that exactly $K$ (out of 21) classifiers will make an error assuming each classifier has an error rate of $\epsilon = 0.3$ and makes its errors independently of the other classifier. The area under the curve for 11 or more classifiers being simultaneously wrong is 0.026 (much less than $\epsilon$).

[Credit: T. G Dietterich, Ensemble Methods in Machine Learning]
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Figure: $\epsilon = 0.3$: (left) $N = 11$ classifiers, (middle) $N = 21$, (right) $N = 121$. 
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Figure: $\epsilon = 0.49$: (left) $N = 11$, (middle) $N = 121$, (right) $N = 10001$. 
Clear demonstration of the power of ensemble methods
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- "Our experience is that most efforts should be concentrated in deriving substantially different approaches, rather than refining a simple technique."
- "We strongly believe that the success of an ensemble approach depends on the ability of its various predictors to expose different complementing aspects of the data. Experience shows that this is very different than optimizing the accuracy of each individual predictor."
BellKor’s Pragmatic Chaos Wins $1 Million Netflix Prize by Mere Minutes

AFTER NEARLY THREE years of late nights and heavy collaboration, a team led by AT&T Research engineers has won the $1 million Netflix Prize for devising the best way to improve the company’s movie recommendation algorithm, which generates an average of 30 billion predictions per day, by 10 percent or more.

Amazingly, the decision came down to a matter of minutes, according to Netflix Prize chief Neil Hunt. BellKor’s Pragmatic Chaos submitted their solution
Bootstrap Estimation

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- Bagging: bootstrap aggregation (Breiman 1994)
FIGURE 8.2. (Top left:) B-spline smooth of data.  
(Top right:) B-spline smooth plus and minus $1.96 \times$ standard error bands.  
(Bottom left:) Ten bootstrap replicates of the B-spline smooth.  
(Bottom right:) B-spline smooth with 95% standard error bands computed from the bootstrap distribution.
Bagging

- Simple idea: generate $M$ bootstrap samples from your original training set.
  - Train on each one to get $y_m$, and average them

$$y_{bag}^M(x) = \frac{1}{M} \sum_{m=1}^{M} y_m(x)$$

For regression: average predictions
For classification: average class probabilities (or take the majority vote if only hard outputs available)

Bagging approximates the Bayesian posterior mean. The more bootstraps the better, so use as many as you have time for.

Each bootstrap sample is drawn with replacement, so each one contains some duplicates of certain training points and leaves out other training points completely.
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Boosting (AdaBoost): Summary

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- Final classifier: weighted sum of component classifiers
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Can you apply this learning module many times to get a strong learner that can get close to zero error rate on the training data?
Making Weak Learners Stronger

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  - That seems like a weak assumption but beware!
- Can you apply this learning module many times to get a strong learner that can get close to zero error rate on the training data?
  - Theorists showed how to do this and it actually led to an effective new learning procedure (Freund & Shapire, 1996).
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  - How do we weight the models in the committee?
How to Train Each Classifier

- Input: $\mathbf{x}$, Output: $y(\mathbf{x}) \in \{1, -1\}$
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- Cost function for classifier \( m \)

\[
J_m = \sum_{n=1}^{N} w_m^n [y_m(x^n) \neq t^n] = \sum \text{weighted errors}
\]

1 if error, 0 o.w.
How to weight each training case for classifier \( m \)

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- The **quality of the classifier** is
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  \alpha_m = \ln \left( \frac{1 - \epsilon_m}{\epsilon_m} \right)
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  It is zero if the classifier has weighted error rate of 0.5 and infinity if the classifier is perfect.
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- The weights for the next round are then
  \[
  w_{n}^{m+1} = \exp \left( -\frac{1}{2} t^{(n)} \sum_{i=1}^{m} \alpha_i y_i(x^{(n)}) \right) = w_n^m \exp \left( -\frac{1}{2} t^{(n)} \alpha_m y_m(x^{(n)}) \right)
  \]
How to make predictions using a committee of classifiers

- Weight the binary prediction of each classifier by the quality of that classifier:

\[ y_M(x) = \text{sign} \left( \sum_{m=1}^{M} \frac{1}{2} \alpha_m y_m(x) \right) \]

- This is how to do inference, i.e., how to compute the prediction for each new example.
AdaBoost Algorithm

- **Input:** $\{x^{(n)}, t^{(n)}\}_{n=1}^{N}$, and **WeakLearn**: learning procedure, produces classifier $y(x)$
- **Initialize example weights:** $w_n^m(x) = 1/N$
- **For** $m=1:M$
  - $y_m(x) = \text{WeakLearn}(\{x\}, t, w)$, fit classifier by minimizing
    \[ J_m = \sum_{n=1}^{N} w_n^m [y_m(x^n) \neq t^{(n)}] \]
  - Compute unnormalized error rate
    \[ \epsilon_m = \frac{J_m}{\sum w_n^m} \]
  - Compute classifier coefficient $\alpha_m = \log \frac{1-\epsilon_m}{\epsilon_m}$
  - Update data weights
    \[ w_n^{m+1} = w_n^m \exp \left( -\frac{1}{2} t^{(n)} \alpha_m y_m(x^{(n)}) \right) \]
- **Final model**
  \[ Y(x) = \text{sign}(y_M(x)) = \text{sign}\left(\sum_{m=1}^{M} \alpha_m y_m(x)\right) \]
AdaBoost Example

- Training data

[Slide credit: Verma & Thrun]
AdaBoost Example

Round 1

\[ h_1 \]

\[ D_2 \]

\[ \varepsilon_1 = 0.30 \]
\[ \alpha_1 = 0.42 \]

[Slide credit: Verma & Thrun]
AdaBoost Example

Round 2

[Slide credit: Verma & Thrun]
AdaBoost Example

Round 3

[Slide credit: Verma & Thrun]
AdaBoost Example

- Final classifier

\[
H_{\text{final}} = \text{sign}(0.42 + 0.65 + 0.92)
\]

[Slide credit: Verma & Thrun]
AdaBoost example

- Each figure shows the number $m$ of base learners trained so far, the decision of the most recent learner (dashed black), and the boundary of the ensemble (green).

AdaBoost Applet: [http://cseweb.ucsd.edu/~yfreund/adaboost/](http://cseweb.ucsd.edu/~yfreund/adaboost/)
An alternative derivation of ADABOOST

- Just write down the right cost function and optimize each parameter to minimize it
  - stagewise additive modeling (Friedman et al. 2000)
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- At each step employ the exponential loss function for classifier $m$

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E = \sum_{n=1}^{N} \exp\{-t^{(n)}f_m(x^{(n)})\}
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- We want to minimize \( E \) w.r.t. \( \alpha_m \) and the parameters of the classifier \( y_m(x) \)
- We do this in a sequential manner, one classifier at a time
- Misclassification: 0/1 loss
- Exponential loss: $\exp(-t \cdot f(x))$ (AdaBoost)
- Squared error: $(t - f(x))^2$
- Soft-margin support vector (hinge loss): $\max(0, 1 - t \cdot y)$
Learning classifier $m$ using exponential loss

- At iteration $m$, the energy is computed as

$$E = \sum_{n=1}^{N} \exp\{ -t^{(n)} f_m(x^{(n)}) \}$$
Learning classifier \( m \) using exponential loss

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We can compute the part that is relevant for the $m$-th classifier

$$E_{\text{relevant}} = \sum_{n=1}^{N} \exp \left( -t^{(n)} f_{m-1}(x^{(n)}) - \frac{1}{2} t^{(n)} \alpha_m y_m(x^{(n)}) \right)$$
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Thus we minimize the weighted number of wrong examples
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= \left( e^{\frac{\alpha m}{2}} - e^{-\frac{\alpha m}{2}} \right) \sum_n w_n^m [t^{(n)} \neq y_m(x^{(n)})] + e^{-\frac{\alpha m}{2}} \sum_n w_n^m
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\[ = \left( e^{\frac{\alpha m}{2}} - e^{-\frac{\alpha m}{2}} \right) \sum_{n} w_n^m [t^{(n)} \neq y_m(x^{(n)})] + e^{-\frac{\alpha m}{2}} \sum_{n} w_n^m \]

\[ = \underbrace{\left( e^{\frac{\alpha m}{2}} - e^{-\frac{\alpha m}{2}} \right)}_{\text{multiplicative constant}} \underbrace{\sum_{n} w_n^m [t^{(n)} \neq y_m(x^{(n)})]}_{\text{wrong cases}} + \underbrace{e^{-\frac{\alpha m}{2}} \sum_{n} w_n^m}_{\text{unmodifiable}} \]

- The second term is constant w.r.t. \( y_m(x) \)
Continuing the derivation

\[ E_{\text{relevant}} = \sum_{n=1}^{N} w_n^m \exp \left( -t(n) \frac{\alpha m}{2} y_m(x^{(n)}) \right) \]

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- The second term is constant w.r.t. \( y_m(x) \)
- Thus we minimize the weighted number of wrong examples
AdaBoost Algorithm

- **Input:** \( \{x^{(n)}, t^{(n)}\}_{n=1}^{N} \), and **WeakLearn**: learning procedure, produces classifier \( y(x) \)
- **Initialize example weights:** \( w_{n}^{m}(x) = 1/N \)
- **For** \( m=1:M \)
  - \( y_{m}(x) = \text{WeakLearn}(\{x\}, t, w) \), fit classifier by minimizing
    \[
    J_{m} = \sum_{n=1}^{N} w_{n}^{m} [y_{m}(x^{n}) \neq t^{(n)}]
    \]
  - Compute unnormalized error rate
    \[
    \epsilon_{m} = \frac{J_{m}}{\sum w_{n}^{m}}
    \]
  - Compute classifier coefficient \( \alpha_{m} = \log \frac{1-\epsilon_{m}}{\epsilon_{m}} \)
  - Update data weights
    \[
    w_{n}^{m+1} = w_{n}^{m} \exp \left( -\frac{1}{2} t^{(n)} \alpha_{m} y_{m}(x^{(n)}) \right)
    \]
- **Final model**
  \[
  Y(x) = \text{sign}(y_{M}(x)) = \text{sign} \left( \sum_{m=1}^{M} \alpha_{m} y_{m}(x) \right)
  \]
An impressive example of boosting

- Viola and Jones created a very fast face detector that can be scanned across a large image to find the faces.
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    - So its easy to evaluate a huge number of base classifiers and they are very fast at runtime.
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  ![Image of face detection]

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  - There is a neat trick for computing the total intensity in a rectangle in a few operations.
    - So it's easy to evaluate a huge number of base classifiers and they are very fast at runtime.
  - The algorithm adds classifiers greedily based on their quality on the weighted training cases.
AdaBoost in Face Detection

- Famous application of boosting: detecting faces in images
- Two twists on standard algorithm
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  - Pre-define weak classifiers, so optimization = selection
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- Two twists on standard algorithm
  - Pre-define weak classifiers, so optimization = selection
  - Change loss function for weak learners: false positives less costly than misses
AdaBoost Face Detection Results