University of Toronto Mississauga
Department of Mathematical and Computational Sciences
CSC 338 — Numerical Methods, Spring 2012

Midterm Test

Name:

Student Number:

Note: Open textbook. No aids allowed other than the course textbook (Heath). 50 minutes. 7 pages. 4 questions. 46 points. Write all answers on the test booklet, using the back of pages if necessary. The last page is blank. Unless otherwise specified, all answers should be justified. Good luck!

If you do not know the answer to a question, and you write "I don't know", you will receive 20% of the marks of that question. If you just leave a question blank with no such statement, you get 0 marks for that question.

1. (12 points total) For each of the following statements, state whether it is true or false without giving an explanation (2 points each):

   (a) A good algorithm will produce an accurate solution regardless of the condition of the problem being solved.

   (b) Floating-point numbers are distributed uniformly through their range.

   (c) If a matrix has a very small determinant, then the matrix is nearly singular.

   (d) The multipliers in Gaussian elimination with partial pivoting are bounded above by 1 in magnitude.

   (e) Fitting a straight line to a set of data points is a linear least squares problem, whereas fitting a quadratic polynomial to the data is a non-linear least squares problem.

   (f) A Householder transformation is orthogonal.

2. (12 points total) Assume a decimal (base 10) floating-point system having 6 digits of precision and an exponent range from -15 to $+15$. Assume the system is normalized and does not have gradual underflow. What is the result of each of the following floating-point arithmetic operations? (2 points each)

   (a) $1 + 10^5$

   (b) $1 + 10^7$

   (c) $1 + 10^{-7}$

   (d) $10^8/10^{-9}$

   (e) $10^{12} + 10^5$

   (f) $10^{-10} \times 10^{-7}$

3. (10 points) Using Gaussian elimination without pivoting, derive the LU factorization of the matrix below. Give L, U and the elementary elimination matrices.

$$\begin{pmatrix} 1 & 1 & 3 \\ 2 & 3 & 4 \\ -2 & -4 & 1 \end{pmatrix}$$

4. (12 points total) Suppose you are fitting a straight line to the three data points $(0, 1)$, $(1, 3)$ and $(3, 3)$. That is, you want to find values x_1 and x_2 so that the line $y(t) = x_1 + x_2 t$ passes as close as possible to all three data points. In particular, you want to minimize the squared error.

   (a) (6 points) Write down the over-determined system of linear equations for the least squares problem (*i.e.*, the linear equations that should be approximately satisfied).

   (b) (3 points) Write down the corresponding normal equations.

(c) (3 points) Write down the corresponding augmented system.

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