NAME (PRINT): _____
Last/Surname First /Given Name

STUDENT #: _____ **SIGNATURE:** _____

UNIVERSITY OF TORONTO MISSISSAUGA
APRIL 2011 FINAL EXAMINATION
CSC338H5S
Numerical Methods
Anthony Bonner
Duration - 2 hours
Aids: The course textbook (Heath), and
one double-sided Letter (8-1/2 x 11) sheet of handwritten notes

The University of Toronto Mississauga and you, as a student, share a commitment to academic integrity. You are reminded that you may be charged with an academic offence for possessing any unauthorized aids during the writing of an exam, including but not limited to any electronic devices with storage, such as cell phones, pagers, personal digital assistants (PDAs), iPods, and MP3 players. Unauthorized calculators and notes are also not permitted. Do not have any of these items in your possession in the area of your desk. Please turn the electronics off and put all unauthorized aids with your belongings at the front of the room before the examination begins. If any of these items are kept with you during the writing of your exam, you may be charged with an academic offence. A typical penalty may cause you to fail the course.

*Please note, you **CANNOT** petition to **re-write** an examination once the exam has begun.*

Write your answers on the examination sheet in the spaces provided. You may use the backs of pages if necessary. Concise, well-written answers will receive more points than long, rambling ones. Unless stated otherwise, all answers should be justified.

If you do not know the answer to a question, and you write "I don't know", you will receive 20% of the marks of that question. If you just leave a question blank with no such statement, you get 0 marks for that question.

This page is for marking purposes only

Question	Value	Score
1	16	
2	5	
3	10	
4	10	
5	10	
6	15	
7	10	
8	10	
9	14	
Total	100	

1. (16 points total) True or False.

   For each of the following statements, state whether it is true or false, without giving a explanation (2 points each):

   (a) The choice of algorithm for solving a problem has no effect on the propagated data error.

   (b) Floating point numbers are distributed uniformly throughout their range.

   (c) If a matrix has a very small determinant, then it has a very high condition number.

   (d) The product of two symmetric matrices is symmetric.

   (e) Multiplication by an orthogonal matrix preserves the Euclidean norm of a vector.

   (f) If the $n \times n$ matrix Q is a Householder transformation and x is an arbitrary n-vector, then the last k components of Qx are zero, for some $k < n$.

   (g) Newton's method of solving equations is an example of a fixed-point iteration scheme.

   (h) In minimizing a unimodal function of one variable by golden section search, the point discarded at each iteration is always the point having the largest function value.

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2. (5 points total) Approximations and Computer Arithmetic.

   If $x \approx y$, then we would expect some cancellation in computing $\log(x) - \log(y)$. On the other hand, $\log(x) - \log(y) = \log(x/y)$, and the latter involves no cancellation. Does this mean that computing $\log(x/y)$ is likely to give a better result? (Hint: for what values is the log function sensitive?)

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3. (10 points total) Systems of Linear Equations.

   Suppose A, B and C are non-singular $n \times n$ matrices, and b is an n-vector. How would you efficiently evaluate the following expression without computing any matrix inverses:

$$(A^{-1} + 5C)(2I - B^{-1})(A^{-1} + 4C)b$$

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4. (10 points total) Linear Least Squares.

   (a) (5 points) Set up the linear least squares system $Ax \approx b$ for fitting the model function $f(t, x) = x_1 + x_2 e^t + x_3 e^{2t}$ to the five data points $(0, 1)$, $(1, 2)$, $(2, 3)$, $(3, 5)$ and $(4, 7)$.

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(b) (5 points) Consider the vector

$$a = \begin{bmatrix} -2 \\ 2 \\ -2 \\ 2 \end{bmatrix}$$

Specify a Householder transformation matrix that annihilates all but the first component of a.

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5. (10 points total) Nonlinear Equations.

   (a) (5 points) What is the convergence rate for Newton's method for finding the root $x = 2$ of the equation $(x-1)(x-2)^2 = 0$?

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(b) (5 points) What is the convergence rate for Newton's method for finding the root $x = 2$ of the equation $(x-1)^2(x-2) = 0$?

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6. (15 points total) Nonlinear Equations.

   We wish to solve the equation $(x-1)^2 - 2 = 0$.

   (a) (5 points) With $x_0 = 2$ as the starting point, what is the value of x_1 if we use Newton's method?

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(b) (5 points) With $x_0 = 2$ and $x_1 = 3$ as the starting points, what is the value of x_2 if we use the secant method?

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(c) (5 points) If we assume the starting guess has an accuracy of 4 bits, how many iterations would be necessary to attain 24-bit accuracy if we use Newton's method?

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7. (10 points total) Optimization.

   Suppose that a real-valued function f is unimodal on the interval $[a, b]$, and x_1 and x_2 are points in the interval such that $x_1 < x_2$ and $f(x_1) < f(x_2)$.

   (a) (5 points) What is the shortest interval in which you know the minimum of f must lie?

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(b) (5 points) How would your answer change if we happened to have $f(x_1) = f(x_2)$.

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8. (10 points total) Optimization.

   We wish to find the minimum of the function $f(x) = x^3/3 - 2x + 1$.

   (a) (5 points) With $x_0 = 1$ as the starting point, what is the value of x_1 if we use Newton's method?

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(b) (5 points) If we assume the starting guess has an accuracy of 2 bits, how many iterations would be necessary to attain 53-bit accuracy if we use Newton's method?

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9. (14 points total) Interpolation.

   Given the three data points $(-1, 1)$, $(0, 0)$ and $(1, 1)$,

   (a) (5 points) Determine the interpolating polynomial using the Lagrange basis.

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(b) (5 points) Determine the interpolating polynomial using the Newton basis.

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(c) (4 points) Show that the two representations give the same polynomial.

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This page is for answers and rough work.

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This page is for answers and rough work.

Total marks = 100 **END OF EXAM**