A simple grammar for Arithmetic Expressions

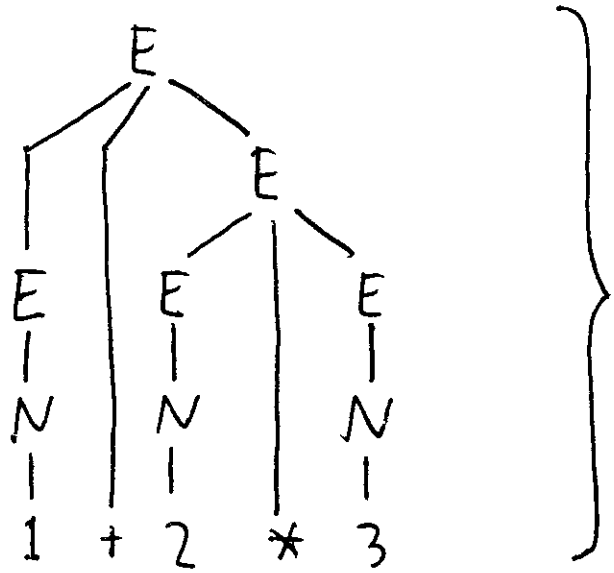
$E \longrightarrow E + E$

$E \longrightarrow E * E$

$E \longrightarrow N$

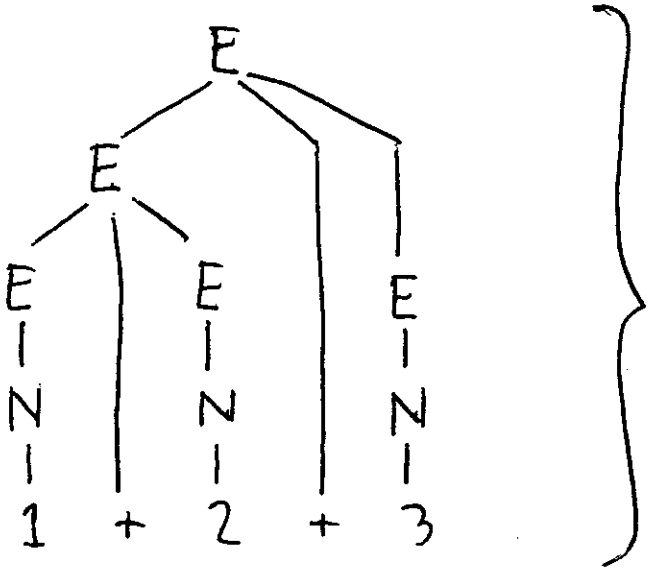
$N \longrightarrow 1 \mid 2 \mid 3 \mid 4 \mid \cdots$

This grammar is <u>ambiguous</u> since the sentence <u>1 + 2 * 3</u> has two parse trees.



corresponds to

1 + (2 * 3)



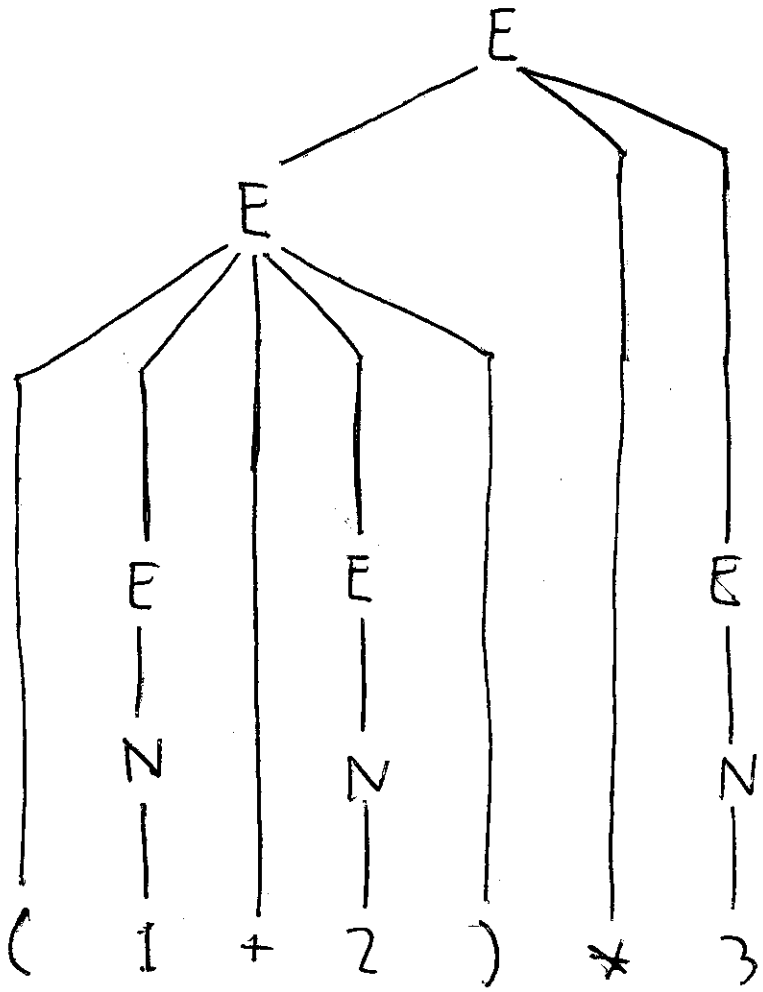
corresponds to

(1 + 2) * 3

We can make the grammar unambiguous by introducing parentheses.

$$E \rightarrow (E + E)$$
$$E \rightarrow E * E$$
$$E \rightarrow N$$
$$N \rightarrow 1 \mid 2 \mid 3 \mid 4 \mid \ldots$$

In this language, $(1 + 2) * 3$ is a sentence, and it has only one parse tree.


E
E
E
N
1
+
E
N
2
)
*
E
N
3
(

Unfortunately although it is unambiguous, this grammar forces us to use lots of brackets with addition. eg,

(1 + 2)
((1 + 2) + 3)
(1 + (2 + 3))
((1+2) + (3 + 4))

} grammatical expressions

1 + 2
1 + 2 + 3
1 + 2 + 3 + 4

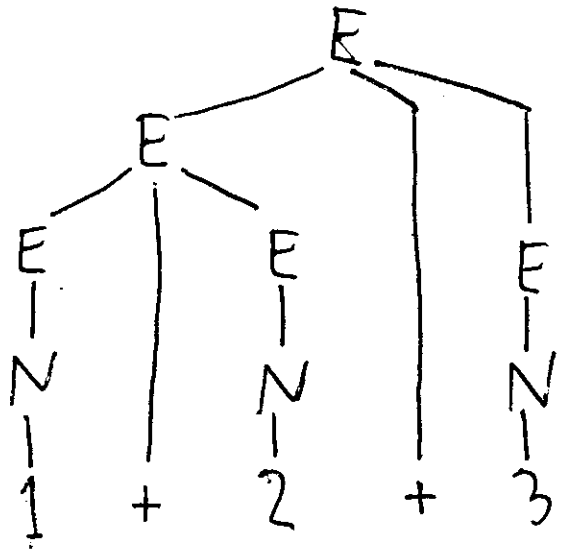
} ungrammatical expressions

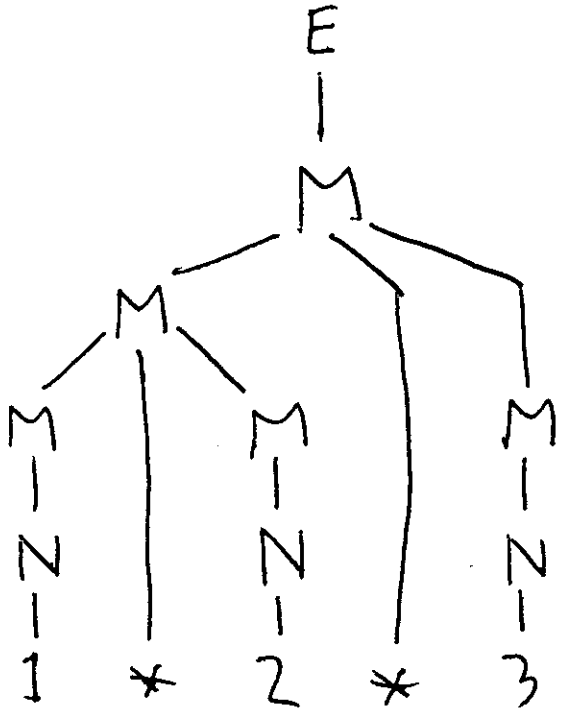
- We can overcome this problem with a grammar that gives multiplication precedence over addition.

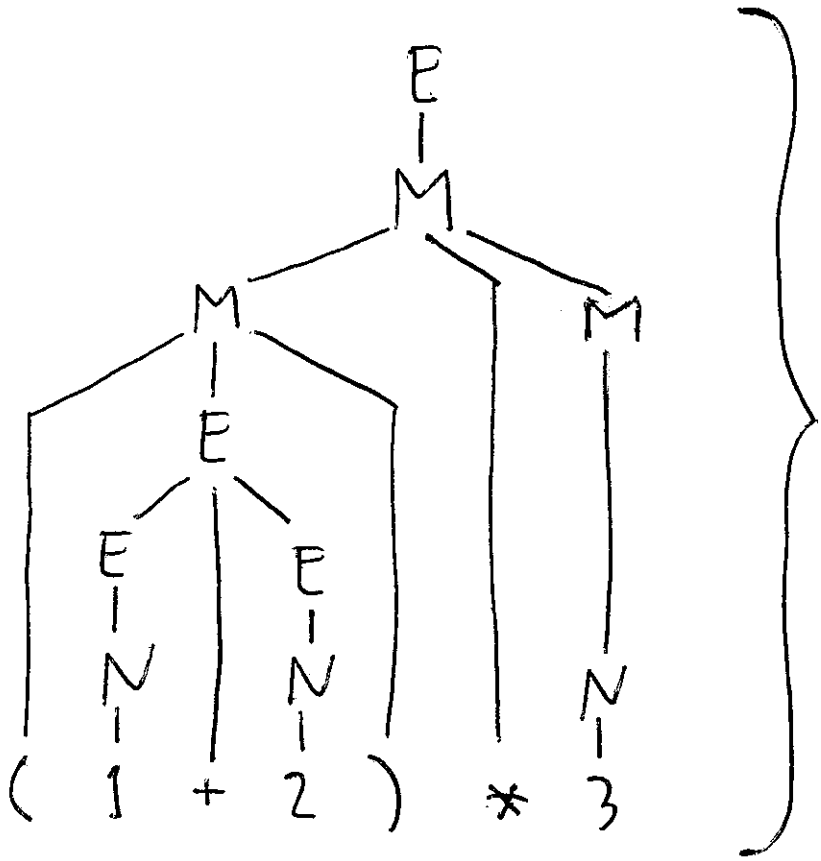
- To do this, we introduce a new non-terminal symbol, M, to represent terms that can be multiplied.

$$E \rightarrow E+E \mid M \mid N$$
$$M \rightarrow M*M \mid (E) \mid N$$
$$N \rightarrow 1 \mid 2 \mid 3 \mid 4 \mid \cdots$$

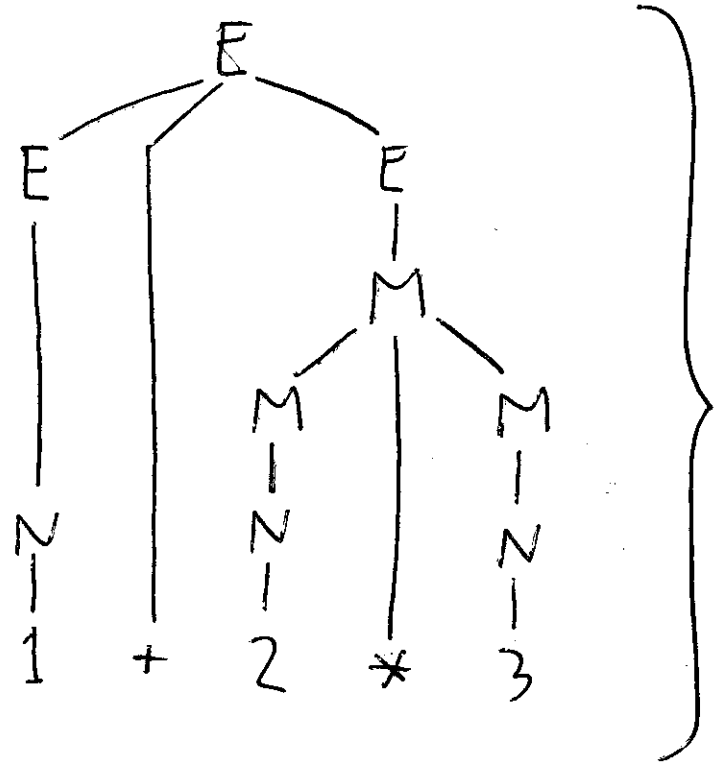
Sample Parses (without brackets)







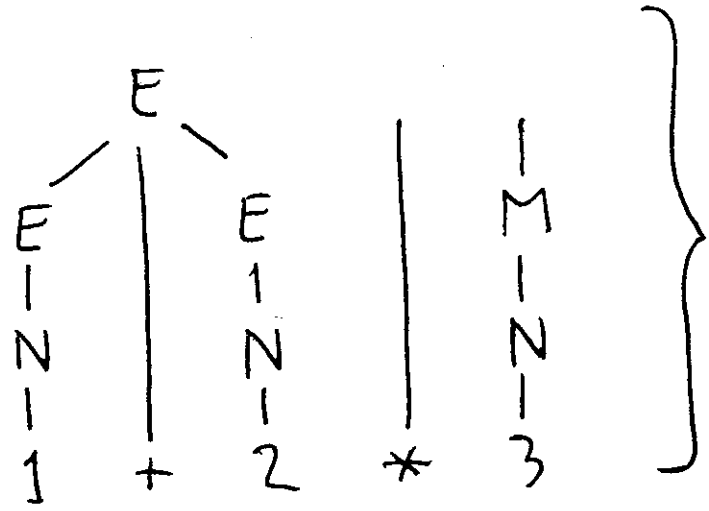
Expressions with brackets



Multiplication takes precedence over addition.

Observation 1

- We cannot parse 1+2*3 as if it meant (1+2)*3.
- If we try, we cannot build a complete parse tree.

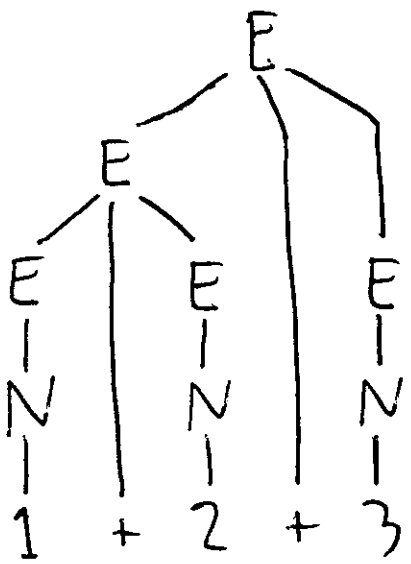


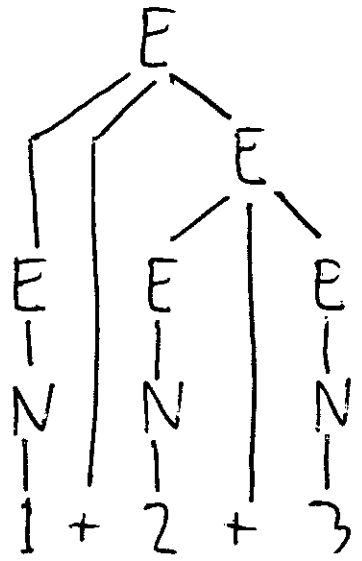
Cannot parse any further.

The grammar has no rules of the form ? → E*M

Observation 2

This grammar is still ambiguous (although in a less serious way) since the sentence 1+2+3 has two parse trees.





ie, The grammar does not tell us in what order to carry out a sequence of additions (or multiplications).

- We can remove this ambiguity by introducing a new non-terminal symbol, E_1, to represent the smallest term in a large addition.

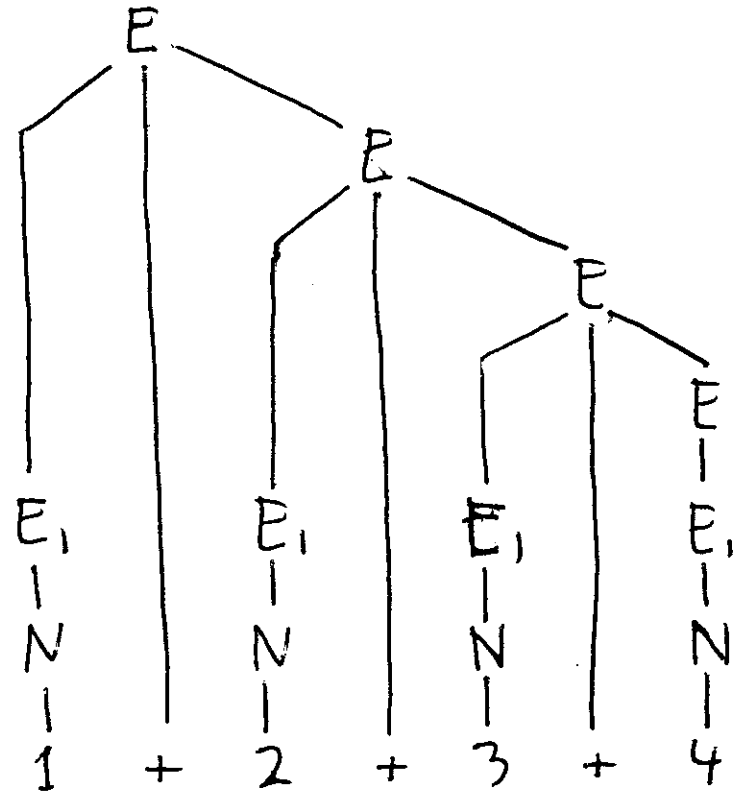
- Likewise, M_1 represents the smallest term in a large multiplication.

$$E \longrightarrow E_1 \mid E_1 + E$$
$$E_1 \longrightarrow M \mid N$$

$$M \longrightarrow M_1 \mid M_1 * M$$
$$M_1 \longrightarrow (E) \mid N$$

$$N \longrightarrow 1 \mid 2 \mid 3 \mid 4 \mid \cdots$$

Parsing Additions

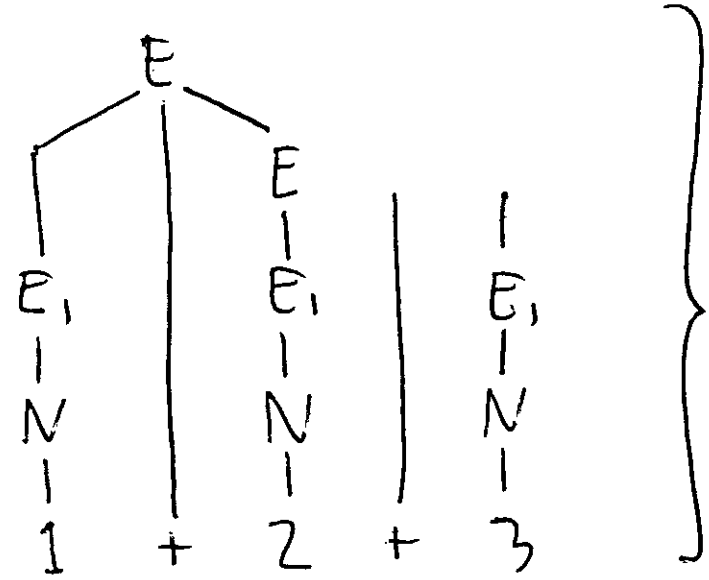


Note: Additions are grouped from left to right.

Observation

If we attempt to group additions from left to right, we cannot build a complete parse tree.

eg,



Can't parse any further

The grammar has no rules of the form

$$? \longrightarrow E + E_1$$