Lecture 5
Decision Trees
some classifiers \((k\text{-}\text{nearest nbor})\) rely on distances between input vectors

what if want to classify food you like to eat?

classifiers for instances represented as lists of attributes (**nominal data**): make decisions based on sets of attribute values

- internal nodes test **attributes**
- branches from node based on attribute **values**
- leaf nodes are outputs (**class**)

instances classified by following path from root to leaf of tree
Decision tree representations

represents a disjunction of conjunctions of constraints on attribute values

if features are continuous, may test value against threshold: $X_1 > 5$?

decision boundaries divide feature space into axis-parallel rectangles

interpretable system:

- understand decision for test example as series of decisions
  (sweet-and-sour pork: $Smell = pungent \land Color = red \rightarrow +$)

- also understand category based on logical descriptions

decision trees can represent any boolean function, but in the worst case requires exponentially many nodes
**Decision Tree Algorithm**

begin with simple, greedy, recursive approach, builds up tree node-by-node

1. pick an attribute to split at a non-terminal node

2. split examples into groups based on attribute value

3. for each group:
   
   (a) if no examples – **return majority from parent**

   (b) else if all examples same class – **return class**

   (c) else if no attribute (non-deterministic domain or errors)

       return majority

   (d) else loop to step 1
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What makes a good tree?

Not too small – need to include enough attributes to handle possibly subtle distinctions in data

Not too big

- computational efficiency (avoid redundant, spurious attributes)
- avoid over-fitting training examples (noisy, scarce data)

Occam’s Razor: find simplest hypothesis (tree) that is consistent with all observations

inductive bias – small trees, with informative nodes near root