CSC410 tutorial: SMT for problem solving

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In this tutorial:

- Uninterpreted Functions: seating arrangement.
Z3 : Linear Integer Arithmetic

- Create integer variables: \(x = \text{Int}(\text{``x''})\)
- Z3 supports all basic arithmetic operations:

```python
x = Int(\text{``x''})
y = Int(\text{``y''})
e1 = x + y
e2 = x - y
e3 = 2 \times x \# \text{multiplication by a constant}
comp = \text{Or}([x > y, x < y, x \leq y, x \neq y, x == y])
```
Job shop problem

- We are given $n$ jobs $J_1, \ldots, J_n$.
- Each job is a set of tasks $J_i = t_{i,1}, \ldots, t_{i,n_i}$ that need to be executed sequentially on a set of $m$ machines.
- Each task has a fixed duration, and a machine can only process a single task at a time. A task started has to be processed until its completion.
Example problem

\[
\begin{align*}
(0,3), & \quad (1,2), \quad (2,2) \quad \# \quad \text{(blue)} \quad \text{job w. 3 tasks on machines 0,1,2} \\
(0,2), & \quad (2,1), \quad (1,4) \quad \# \quad \text{(red)} \\
(1,4), & \quad (2,3) \quad \# \quad \text{(green)}
\end{align*}
\]

A solution:

See `test_input/` for input examples.
Using Z3 for the job shop problem.

- Variables?
Using Z3 for the job shop problem.

- Variables?
- Constraints?
  - Tasks start at positive time.
Using Z3 for the job shop problem.

- Variables?
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  - Tasks start at positive time.
  - Tasks in one job have to be executed sequentially.
Using Z3 for the job shop problem.

- Variables?
- Constraints?
  - Tasks start at positive time.
  - Tasks in one job have to be executed sequentially.
  - Tasks on one machine have to be executed sequentially.
Previous solution satisfies constraints, but not optimal.
Optimization

Previous solution satisfies constraints, but not optimal.

• Z3 has an optimization solver!

```python
s = Optimize()  # instead of Solver()
s.add(c)  # add some clause.
s.minimize(..)  # add some minimization objective.
s.maximize(..)  # add some maximization objective.
s.check()  # check sat + minimize/maximize objectives
s.model()
```
Uninterpreted functions

- Just another kind of variable.
- Declare uninterpreted function from integer to integer or boolean:

```
f = Function(``f'`, IntSort(), IntSort())
f2 = Function (``f2'`, IntSort(), BoolSort())
```

.. or any Z3 sort to Z3 sort.
- Function application syntax

```
x = Int(``x'')
b = f2(f(x)) # b is a boolean
```
Finding a seating arrangement

At the wedding reception, there are five guests, Colin, Emily, Kate, Fred, and Irene, who are not sure where to sit at the dinner table. They ask the bride’s mother, who responds: “As I remember, Colin is not next to Kate, Emily is not next to Fred or Kate. Neither Kate or Emily are next to Irene. And Fred should sit on Irene’s left.”

As you look at them from the opposite side of the table, can you correctly seat the guests from left to right?
Finding a seating arrangement

"As I remember, Colin is not next to Kate, Emily is not next to Fred or Kate. Neither Kate or Emily are next to Irene. And Fred should sit on Irene’s left."

Need only one variable: uninterpreted function position from integer (guest id) to integer (position).
Thank you!

Questions?