1 Equivalencies

Prove the following equivalencies by using the formal semantic of the linear temporal logic (LTL):

• $\neg \lozenge \varphi \equiv \square \neg \varphi$
  
  Proof:
  
  $\sigma \models \neg \lozenge \varphi$
  $\sigma \lozenge \varphi$
  $\neg \exists j : \sigma[j...] \models \varphi$
  $\forall j : \sigma[j...] \varphi$
  $\forall j : \sigma[j...] \models \neg \varphi$
  $\sigma \models \square \neg \varphi$

• $\varphi \cup \psi \equiv \psi \lor (\varphi \land \Box (\varphi \lor \psi))$
  
  Proof:
  
  $\sigma \models \varphi \cup \psi$
  $\exists j : \sigma[j...] \models \psi$ and $\forall i < j : \sigma[i...] \models \varphi$
  if $j = 0$:
  
  $\sigma \models \psi$
  $\sigma \models \psi \lor (\varphi \land \Box (\varphi \lor \psi))$
  if $j > 0$:
  
  $\exists j > 0 : \sigma[j...] \models \psi$ and $\forall i < j : \sigma[i...] \models \varphi$
  $\exists j' : \sigma[j' + 1...] \models \psi$ and $\forall i < j' : \sigma[i + 1...] \models \varphi$ and $\sigma \models \varphi$
  $\sigma[1...] \models \varphi \cup \psi$ and $\sigma \models \varphi$
  $\sigma \models \Box (\varphi \lor \psi)$ and $\sigma \models \varphi$
  $\sigma \models \Box (\varphi \lor \psi) \land \varphi$
  $\sigma \models \psi \lor (\varphi \land \Box (\varphi \lor \psi))$

• $\Box (\varphi \cup \psi) \equiv (\Box \varphi) \lor (\Box \psi)$
  
  Proof:
  
  $\sigma \models \Box (\varphi \cup \psi)$
  $\sigma[1...] \models \varphi \cup \psi$
  $\exists j : \sigma[j + 1...] \models \psi$ and $\forall i < j : \sigma[i + 1...] \models \varphi$
  $\exists j : \sigma[j...] \models \Box \psi$ and $\forall i < j : \sigma[i...] \models \Box \varphi$
  $\sigma \models (\Box \varphi) \lor (\Box \psi)$

• $\Diamond \Box \Diamond \varphi \equiv \Box \Diamond \varphi$
2 Formalizing Descriptions

Write LTL formulas for these English specifications. Make sure that a response to a request cannot be issued in the same state that the request is issued first and has to be delayed by at least one step.

- No persisting request will be left unanswered:
  $$\Diamond \Box req \rightarrow \Box \Diamond act$$

- No request will be left unanswered.
  $$\neg \Diamond (req \land \neg \Diamond act)$$

- Each request has exactly one answer and there are no overlapping requests.

Come up with LTL formulas for these specifications that seem to reference events from previous steps.

- Every activation should happen immediately after a warning.
  $$\Box (\Diamond act \rightarrow wrn)$$

- Every activation should be preceded by a dedicated warning.
  $$\neg (\neg wrn \cup act) \land \Box (act \rightarrow (wrn \lor \Box (\neg wrn \cup act)))$$

- Overload signal should indicate uninterrupted action up until that point.
  $$\neg \Diamond (\neg act \land \Diamond ovr)$$