THE PUMPING LEMMA
Theorem. For any regular language $L$ there exists an integer $n$, such that for all $x \in L$ with $|x| \geq n$, there exist $u, v, w \in \Sigma^*$, such that

1. $x = uvw$
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$$x = \begin{array}{l}
uw \in L \\
uvw \in L \\
uvvv \in L \\
\vdots
\end{array}$$
USING PUMPING LEMMA TO PROVE NON-REGULARITY

\[ L \text{ regular} \implies L \text{ satisfies P.L.} \]
\[ L \text{ non-regular} \implies ? \]
\[ L \text{ non-regular} \iff L \text{ doesn’t satisfy P.L.} \]

Negation:
\[ \exists n \in \mathbb{N} \forall x \in L \text{ with } |x| \geq n \]
\[ \exists u, v, w \in \Sigma^* \]
all of these hold:

1. \[ x = uvw \]
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Using Pumping Lemma to prove non-regularity

$L$ regular $\implies L$ satisfies P.L.

$L$ non-regular $\implies ?$

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$\forall\ n \in \mathbb{N} \ \exists\ x \in L \ \text{with} \ |x| \geq n$
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not all of these hold:
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Using Pumping Lemma to Prove Non-regularity

L regular $\implies$ L satisfies P.L.
L non-regular $\implies$ ?
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Equivalently:

1. $x = uvw$
2. $|uv| \leq n$
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4. $\forall \ i \geq 0: uv^iw \in L$

where not(4) is:

$\exists \ i : uv^iw \notin L$
EXAMPLE 1

Prove that $L = \{0^i1^i : i \geq 0\}$ is NOT regular.
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\[ s + t \leq n, \quad t \geq 1, \quad p \geq 0, \quad s + t + p = n. \]
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$$uv^0w = uw = 0^s0^p1^n = 0^{s+p}1^n \notin L, \quad \text{since } s + p \neq n$$
∃u, v, w such that

(1) \( x = uvw \)

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If (1), (2), (3) hold then (4) fails: it is not the case that for all \( i \), \( uv^i w \) is in \( L \).

In particular, let \( i = 0 \). \( uw \not\in L \).
EXAMPLE 2

Prove that $L = \{0^i : i \text{ is a prime}\}$ is NOT regular.
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If $0^m$ is written as $0^m = uvw$, then $0^m = 0^{|u|}0^{|v|}0^{|w|}$. 
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If $0^m$ is written as $0^m = uvw$, then $0^m = 0|u|0|v|0|w|$
If $|uv| \leq n$ and $|v| \geq 1$, then consider $i = |v| + |w|$:
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If \( |uv| \leq n \) and \( |v| \geq 1 \), then consider \( i = |v| + |w| \):

\[
uv^i w = 0|v|0|v|(|v|+|w|)0|w| = 0(|v|+1)(|v|+|w|) \notin L
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If $|uv| \leq n$ and $|v| \geq 1$, then consider $i = |v| + |w|:

$$uv^i w = 0^{|v|}0^{|v|(|v|+|w|)}0^{|w|}$$

$$= 0(|v|+1)(|v|+|w|) \not\in L$$

Both factors $\geq 2$
EXAMPLE 3

Prove that $L = \{yy : y \in \{0, 1\}^*\}$ is NOT regular.
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If $L$ is regular, then by P.L. $\exists n$ such that $\ldots$
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If $L$ is regular, then by P.L. $\exists n$ such that … 
Let us consider $x = 0^n0^n \in L$. Obviously $|x| \geq n$. 
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If \( L \) is regular, then by P.L. \( \exists n \) such that …
Let us consider \( x = 0^n0^n \in L \). Obviously \( |x| \geq n \).
Can \( 0^n0^n \) be written as \( 0^n0^n = uvw \) such that \( |uv| \leq n \ |v| \geq 1 \) and that for all \( i: uv^i w \in L \)?
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If $L$ is regular, then by P.L. $\exists n$ such that ...

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Can $0^n0^n$ be written as $0^n0^n = uvw$ such that $|uv| \leq n$ $|v| \geq 1$ and that for all $i$: $uv^i w \in L$?

YES! Let $u = \epsilon$, $v = 00$, and $w = 0^{2n-2}$. 
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YES! Let $u = \epsilon$, $v = 00$, and $w = 0^{2n-2}$.
Then $\forall i$, $uv^iw$ is of the form $0^{2k} = 0^k0^k$. 
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Does this mean that \( L \) is regular?
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Does this mean that \( L \) is regular?

**NO.** We have chosen a bad string \( x \). To show that \( L \) fails the P.L., we only need to exhibit some \( x \) that cannot be “pumped” (and \( |x| \geq n \)).
EXAMPLE 3, 2ND ATTEMPT

Prove that $L = \{yy : y \in \{0, 1\}^*\}$ is NOT regular.
EXAMPLE 3, 2ND ATTEMPT

Prove that \( L = \{yy : y \in \{0, 1\}^*\} \) is NOT regular.

Given \( n \) from the P.L., let \( x = (01)^n(01)^n \). Obviously \( x \in L \) and \( |x| \geq n \).
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Q: Can $x$ be “pumped” for some choice of $u, v, w$ with $|uv| \leq n$ and $|v| \geq 1$?

A: Yes! Take $u = \epsilon$, $v = 0101$, $w = (01)^{2n-2}$. 
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A: Yes! Take $u = \epsilon$, $v = 0101$, $w = (01)^{2n-2}$.

Another bad choice of $x$!
EXAMPLE 3, 3RD ATTEMPT

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\[
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\]

Since \( |v| \) is at least 1, this is clearly not of the form \( yy \).